

Math 201
Midterm 1
October 13, 2015

Your name: _____ Your student ID: _____

Circle your professor's name: Hambrook Mkrtchyan Neuman

Sign the honor pledge: I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.: _____

- You should give clear explanations for all your solutions. Simply giving the answer will not earn full credit.
- No calculators are allowed on this exam, but you are allowed one sheet of paper with writing on both sides.
- The notation AB and $A \cap B$ both mean intersection, and you can use them interchangeably.
- You need not reduce such expressions as $\binom{100}{30}$ and $50!$ to a number.

1. (12 points)

A bookshelf contains 4 math books, 6 music books and 7 history books. The books are arranged in a random order and all arrangements are equally likely.

(a)(4 points) What is the total number of possible arrangements?

(b)(8 points) What is the probability that all books in each category are next to each other? Explain your answer.

2. (11 points) A farmer is plowing a square field of side length 2 miles when he suddenly realizes he has only enough gas left to drive 1 mile. The farmer has extra gas in each corner of the field. Assuming that the current location of the farmer is uniformly distributed in the field, what is the probability that he has enough gas left to reach a corner?

3. (11 points) Assume the events A and B are independent. Prove that A^c and B are independent.

4. (11 points) Let X be a random variable with probability mass function

$$p_X(k) = \begin{cases} \frac{1}{9} & \text{if } k = -4, -3, -2, -1, 0, 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases} .$$

Find the probability mass function of the random variable $Y = X^2$.

5. (11 points) You have 3 identical-looking urns. Urn 1 contains 3 red balls. Urn 2 contains 3 green balls. Urn 3 contains 3 blue balls. Pick an urn uniformly at random and add a green ball to it. Then draw a ball from that same urn. If you draw a green ball, what is the probability the other balls in the urn are green?

6. (11 points) There are 10 balls in the basket: 5 of them are blue, 3 are red and 2 are green. 5 balls are drawn at random without replacement. What is the probability that of those five drawn there are two blue, two red and one green?

7. (11 points)

Let X and Y be independent random variables such that $X \sim \text{Geom}(1/3)$ and $Y \sim \text{Geom}(1/4)$. Define the random variable Z to be the minimum of X and Y , that is $Z = \min\{X, Y\}$. What is the probability that Z is larger than 4?

8. (11 points) Which one of the following functions cannot be a cumulative distribution function? Explain your answer.

$$\text{(a)} \quad F(x) = \begin{cases} 1 - e^{-3x}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases}$$

$$\text{(b)} \quad F(x) = \begin{cases} 0, & \text{if } x < 0, \\ \frac{10x-x^2}{25}, & \text{if } 0 \leq x < 10, \\ 1, & \text{if } x \geq 10. \end{cases}$$

$$\text{(c)} \quad F(x) = \begin{cases} 0, & \text{if } x < 0, \\ \frac{1}{3}, & \text{if } 0 \leq x < 2, \\ \frac{1}{2}, & \text{if } 2 \leq x < 4, \\ 1, & \text{if } x \geq 4. \end{cases}$$

9. (11 points) Bob, Mary and Jane's mother makes 6 sandwiches for them and give 2 sandwiches to each child. Suppose there are 2 sandwiches with grape jam, 2 with apricot jam and 2 with peach jam. What is the probability that no child got 2 sandwiches of the same kind, if all ways of distributing the sandwiches are equally likely?