MATH 201: Written Homework 6
Due Wednesday, 6/12 by 1pm EDT

(P1) Throughout this problem, assume that we are playing poker with a standard 52-card deck and each hand there are 5 cards randomly dealt to you. A hand containing four cards that match (same number) is called a *four-of-a-kind*. A hand containing three cards that match, with two other cards that match each other, is called a *full house*. A hand containing three cards that match but is not a four-of-a-kind nor a full-house is called a *three-of-a-kind*. For example, 4 queens and 1 nine is a four-of-a-kind. 3 queens and 2 nines is a full-house. 3 queens, 1 nine, and 1 eight is a three-of-a-kind.

(a) If you are playing poker with a standard 52-card deck where 5 random cards are dealt to you, find the probability that you will get a four-of-a-kind.

(b) If you are playing poker with a standard 52-card deck where 5 random cards are dealt to you, find the probability that you will get a three-of-a-kind.

(c) Estimate the probability that out of 10,000 poker hands (of 5 cards) we will see more than 3 four-of-a-kinds. Use either the normal or the Poisson approximation, whichever is appropriate. Justify your choice of approximation. What can be said about the error in your calculation?

(d) Estimate the probability that out of 10,000 poker hands (of 5 cards) we will see more than 400 three-of-a-kinds. Use either the normal or the Poisson approximation, whichever is appropriate. Justify your choice of approximation. What can be said about the error in your calculation?

(P2) You install 2 new light bulbs: a 60 watt bulb and a 100 watt bulb. Let X be the lifetime of the 60 watt bulb and Y the lifetime of the 100 watt bulb. The package says that the expected lifetime of the 60 watt bulb is an exponential random variable with mean 200 days while the lifetime of the 100 watt bulb is an exponential random variable with mean 100 days.

(a) Write down the PDFs of X and Y.

(b) Compute the CDFs of X and Y.

(c) If f_X is the PDF of X and F_Y is the CDF of Y then

$$P(X < Y) = \int_0^\infty f_X(x)(1 - F_Y(x))dx.$$

Use the above equation and your answers to parts (a) and (b) of this problem to compute $P(X < Y)$.

(d) Now suppose both light bulbs have been running for 10 days straight. How does this change $P(X < Y)$?

(P3) Suppose that $X \sim \text{Geom}(p)$ and $Y \sim \text{Geom}(r)$ are independent. Find the probability $P(X < Y)$.