MATH 201: Written Homework 5
Due Wednesday, 6/5 by 1pm EDT

(P1) Let $Z \sim \mathcal{N}(0,1)$ and $X \sim \mathcal{N}(\mu, \sigma^2)$. This means that Z is a standard normal random variable with mean 0 and variance 1, while X is a normal random variable with mean μ and variance σ^2.

(a) Use integration by parts to show the reduction formula:

$$\int_{-\infty}^{\infty} x^n e^{-x^2/2}\, dx \;=\; \int_{-\infty}^{\infty} (n-1)x^{n-2} e^{-x^2/2}\, dx \quad \text{for} \quad n \geq 2.$$

(b) Calculate $E[Z^3]$, the third moment of Z, using the reduction formula from part (a) with $E[Z] = 0$.

(c) Calculate $E[X^3]$. **Hint:** Do not integrate with the density function of X unless you like messy integration. Instead use the fact that $X = \sigma Z + \mu$ and expand the cube inside the expectation.

(P2) Complete parts (a) and (b) below.

(a) PollsRule.com is sampling likely voters in Nevada to ask if they intend to vote for Joe Biden in the upcoming election. How large must a random sample be in order to be at least 92% certain that the fraction $\hat{p}$ of positive answers in the sample (i.e. those that responded that they will vote for Biden) is within 0.04 of the true fraction p of voters in Nevada that support Biden.

(b) PollsRule.com conducted a poll of 1000 likely voters in Pennsylvania before the 2016 election. Of the 1000 people sampled, 510 preferred Hillary Clinton. What is the probability of seeing such a poll, given that Clinton received 47.5% of the vote in Pennsylvania? Make some reasonable assumptions and use the continuity correction to compute your answer.

(P3) Suppose X is an random variable with mean -3 and variance 4. Compute

(a) $\text{Var}(2X + 10)$

(b) $E[(5X - 7)^2]$