MATH 201: Written Homework 3
Due Wednesday, 5/29 at 1PM EDT

(P1) There are two boxes labeled 1 and 2. Box 1 contains one marble which is equally likely to be red or blue. Box 2 also contains 1 marble which is equally likely to be red or blue. Note: For this problem it is helpful to define some events, for instance $R_j = \{\text{the marble in box } j \text{ is red}\}$.

(a) What is the probability that both marbles are the same color?

(b) Someone randomly chooses a box and opens it. If the marble is red, what is the probability that both marbles are red?

(c) Suppose instead the person selects which marble to reveal as follows: they fist peek in both boxes. If at least one of the marbles is red, then they will definitely reveal a red marble. When this rule does not determine which box to open, then they are equally likely to reveal either one. What is the probability that both marbles are red, given that the revealed marble is red?

(P2) Show that if $X \sim \text{Geom}(p)$ then

$$P(X = n + k | X > n) = P(X = k), \text{ for every } n, k \geq 1.$$

This is called the *memoryless property* of the geometric distribution. It says that if there are no successes in the first n trials then the probability that the first success at trial $n + k$ is the same as the probability that a freshly started sequence of trials yields the first success at trial k. The first n trials are forgotten.

(P3) Let X and Y be independent random variables with

$$P(X = 1) = P(Y = 1) = P(X = -1) = P(Y = -1) = \frac{1}{2}.$$

(a) Are X and XY independent?

(b) Are Y and XY independent?

(c) Are X, Y and XY independent?