

MATH 201: Written Homework 2
Due Friday 5/24

(P1) May and Tim are playing paintball in opposite teams and they try to shoot each other simultaneously.

- The probability that both May and Tim hit their target in a single shot is $1/8$.
- The probability that May hits her target but Tim misses his target is $3/8$.
- The probability that May misses her target but Tim hits his target is $1/8$.
- The probability that both May and Tim miss their target is $3/8$.

They continue to shoot simultaneously until the game ends when one of the two players hits the other. If the two players hit each other on the same time they both lose.

- (a) What is the probability that the battle will end on the n -th round (with the n -th shot of May or Tim)?
- (b) What is the probability that May wins the game ?
- (c) What is the probability that Tim wins the game ?
- (d) Show that the game must end eventually (i.e. show the probability it ends eventually is 1). What is the probability that they both lose the game?

(P2) A crime has been committed in a town of 200,000 inhabitants. The police are looking for a single perpetrator, believed to live in town. DNA evidence is found on the crime scene. Bob's DNA matches the DNA recovered from the crime scene. According to DNA experts, the probability that a random person's DNA matches the crime scene DNA is 1 in 5,000. Before the DNA evidence, Bob was no more likely to be the guilty person than any other person in town.

- (a) What is the probability that Bob is guilty after the DNA evidence appeared? You may assume that if the perpetrator's DNA is tested then it will match the crime scene DNA 100% of the time.

Hint: Reason as in example 2.14 in the textbook.

- (b) Suppose a new method is developed to test DNA. With this new method the probability that a random person's DNA matches the crime scene DNA is now 1 in 30,000. Suppose that with the new method it is confirmed that Bob's DNA matches the DNA recovered from the crime scene. What is the probability that Bob is guilty now?
- (c) Suppose that a witness observed an orange car fleeing the scene of the crime but could not give any more details about the car. The town has 2,000 orange cars. In questioning it is revealed that Bob also drives an orange car. What is the probability that Bob is guilty now if the original method for testing DNA is used? What if the new more accurate method is used?

(P3) Imagine a game of 3 players where exactly one player wins in the end and all players have equal chances of being the winner. The game is repeated six times. Find the probability that there is at least one person who wins no games. **Hint:** Consider the events A_i in which person i wins no games and use the inclusion-exclusion formula.