Math 165

Midterm 2

Nov 7, 2023

Name: ______________________________

UR ID: ______________________________

PLEASE COPY THE HONOR PLEDGE AND SIGN:
(Cursive is not required).
I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

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YOUR SIGNATURE:______________________________________

1. (20 pts) Determine whether each given set S is a subspace of the given vector space V. If so, give a proof; if not, state a property it fails to satisfy.

(a) $V = \mathbb{R}^3$, and $S = \{(a_1, a_2, a_3) \in \mathbb{R}^3 | a_1 = 3a_2 \text{ and } a_3 = -a_2\}$.

Circle final answer. S is a subspace: YES or NO

(b) $V = \mathbb{R}^3$, and $S = \{(a_1, a_2, a_3) \in \mathbb{R}^3 | 5a_1^2 - 3a_2^2 + 6a_3^2 = 0\}$.

Circle final answer. S is a subspace: YES or NO

(c) $V = M_{2\times 2}(\mathbb{R})$, and $S = \left\{ A \in M_{2\times 2}(\mathbb{R}) \;\middle|\; \operatorname{tr}(A) = 0 \right\}$.

Circle final answer. S is a subspace: YES or NO

(d) $V = P_3(\mathbb{R})$, and $S = \left\{ p(x) \in P_3(\mathbb{R}) \;\middle|\; p'(1) = p(0) \right\}$.

Circle final answer. S is a subspace: YES or NO

2. (16 pts)

(a) Use any method to find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 0 & 5 & -4 \end{bmatrix}.$$

Include all details.

Inverse:

(b) Let $\vec{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\vec{a}_2 = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, $\vec{a}_3 = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$, and $\vec{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$. Determine if $\vec{b} \in \text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ or not. If so, write $\vec{b}$ explicitly as a linear combination of $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$; if not, explain why not.

Circle answer. $\vec{b} \in \text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$**: YES or NO**
If answered YES, write $\vec{b}$ as linear combination here:

3. (14 pts) Show that $\{3x^2 + x + 1, 2x + 1, 2\}$ is a basis for $P_2(\mathbb{R})$.

4. (19 pts)

(a) Compute the determinant of the matrix A, defined by

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

$\mathbf{det(A)} =$

(b) Answer the following questions regarding matrix A from part (a).

Circle answer. A **is invertible: YES or NO**
Explanation:

(c) Suppose B is a 3×3 matrix with determinant 2. C is obtained from B by interchanging two rows. D is obtained from B by adding 5 times row 1 to row 2 (while leaving row 1 unchanged). Find the following determinants (no work needed - just put the final answers alone in the answer boxes. There is space for optional work below the answer boxes.)

$\mathbf{det(B^3)} =$	$\mathbf{det(B^T)} =$
$\mathbf{det(B^{-1})} =$	$\mathbf{det(2B)} =$
$\mathbf{det(C)} =$	$\mathbf{det(D)} =$

(d) Suppose K is a $m \times n$ matrix of nullity 5 and where Col(K), the column space of K, has a basis given by the vectors $v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ 1 \\ 5 \\ 7 \end{bmatrix}$. State what m and n have to be. (No work needed though there is space for optional work below the answer boxes)

$\mathbf{m} =$	$\mathbf{n} =$

5. (16 pts) The matrix

$$A = \begin{pmatrix} 1 & -2 & 1 & 3 \\ 3 & -6 & 2 & 7 \\ 4 & -8 & 3 & 10 \end{pmatrix}$$

has reduced row echelon form given by

$$U = \begin{pmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Find a basis for the nullspace of A.

Basis for nullspace:

(b) Find a basis for the column space of A.

Basis for column space:

(c) Compute the rank and nullity of A.

Rank of A is:

Nullity of A is:

6. (15 pts) Choose the correct answer (which should be universally correct) out of the list provided. You do not need to show work, and partial credit will not be offered.

(a) Let $S := \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ be a set of vectors in $\mathbb{R}^n$ for $n \geq 2$. Suppose A is the $n \times n$ matrix given by $A := [\vec{v}_1\ \vec{v}_2\ \ldots\ \vec{v}_n]$, where $\det(A) = 0$. Then $\vec{v}_1$ can always be written as a linear combination of $\vec{v}_2, \ldots, \vec{v}_n$.

- ☐ Statement (a) is true.
- ☐ Statement (a) is false.

(b) Let $A \in M_{m \times n}(\mathbb{R})$ and suppose $1 \leq m < n$. Then:

- ☐ $\dim(nullspace(A)) \leq n - m$.
- ☐ $\dim(nullspace(A)) = n - m$.
- ☐ $\dim(nullspace(A)) \geq n - m$.
- ☐ None of the above conclusions can be drawn without more information.

(c) If A is a lower-triangular 2×2 matrix with entries in $\mathbb{R}$ and $tr(A) = 0$, then:

- ☐ $det(A) \geq 0$.
- ☐ $det(A) = 0$.
- ☐ $det(A) \leq 0$.
- ☐ None of the above conclusions can be drawn without more information.

(d) Let $S := \{p(x) \in P_3(\mathbb{R}) \mid p(x) = p(-x)\}$. Then:

- ☐ S is not a subspace of $P_3(\mathbb{R})$.
- ☐ S is a subspace of $P_3(\mathbb{R})$ with dimension 4.
- ☐ S is a subspace of $P_3(\mathbb{R})$ with dimension 3.
- ☐ S is a subspace of $P_3(\mathbb{R})$ with dimension 2.
- ☐ S is a subspace of $P_3(\mathbb{R})$ with dimension 1.

(e) If A is an $n \times n$ invertible matrix, then $A + A^T$ is also invertible.

- ☐ True
- ☐ False

EXTRA PAGE. You may use this page if you run out of space. Be sure to label your problems on this page and also include a note on the original page telling the graders to look for your work here.