MTH 165: Linear Algebra with Differential Equations

Midterm 2

November 22, 2016

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Indicate your instructor with a check in the box:

Bobkova	MWF 10:25-11:15	
Lubkin	MWF 9:00-9:50	
Rice	TR 14:00-15:15	
Vidaurre	MW 14:00-15:15	

- You have 75 minutes to work on this exam.
- No calculators, cell phones, other electronic devices, books, or notes are allowed during this exam.
- Show all your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You are responsible for checking that this exam has all 11 pages.

QUESTION	VALUE	SCORE
1	21	
2	18	
3	21	
4	20	
5	20	
TOTAL	100	

1. (21 points) Determine whether each given set S is a subspace of the given vector space V. If so, give a proof; if not, provide a counterexample.

(a) $V = P_2(\mathbb{R})$, the set of polynomials of degree at most 2, and $S = \{p \in V : p'(0) = 1\}$.

(b) $V = M_2(\mathbb{R})$, the set of 2×2 matrices, and

$$S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in V \mid a - 4b = c - 5d \right\}.$$

(c) $V = \mathbb{R}^2$, and $S = \{(x, y) \in V : |y| = |x|\}$.

2. (18 points) Answer the following questions, with justification, about a given collection of vectors in a given vector space.

(a) Do the polynomials $p_1(t) = 1 + t^2$, $p_2(t) = t^3$, and $p_3(t) = 4 - t$ span all of $V = P_3(\mathbb{R})$, the set of polynomials of degree at most 3?

(b) Are the functions $f(t) = e^t$, $g(t) = t^2$, and $h(t) = \sin(t)$ linearly independent in $V = \mathcal{C}^2(\mathbb{R})$, the set of functions with everywhere-continuous second derivatives?

(c) Do the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 5 \\ 7 \\ 1 \end{bmatrix}$$

form a basis for $V = \mathbb{R}^3$?

3. (21 points) Let

$$A = \begin{bmatrix} 1 & 2 & 7 & 9 \\ 3 & 7 & 26 & 28 \\ 5 & 11 & 40 & 46 \end{bmatrix}$$

(a) Determine a basis for the row space of A.

(b) Determine a basis for the column space of A.

(c) Determine a basis for the nullspace of A.

4. (20 points) Answer the following about a 31×14 matrix A (that is, a matrix with 31 rows and 14 columns) with $\text{rank}(A) = 14$. No justification is required for parts (a)-(c).

(a) $\text{rowspace}(A)$ is a ______-dimensional subspace of $\mathbb{R}^d$ with $d =$______

(b) $\text{colspace}(A)$ is a ______-dimensional subspace of $\mathbb{R}^d$ with $d =$______

(c) $\text{null}(A)$ is a ______-dimensional subspace of $\mathbb{R}^d$ with $d =$______

(d) Are the rows of A linearly independent? Why or why not?

(e) Are the columns of A linearly independent? Why or why not?

5. (20 points) Answer the following, with justification, about the function

$$T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$$

defined by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a & a+d \\ b+c & a+b+c+d \end{bmatrix}.$$

(a) Show that T is a linear transformation.

(b) Is $A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$ in the kernel of T?

(c) Is $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ in the range of T?

(d) What is $\dim(\ker(T)) + \dim(\mathrm{Rng}(T))$?