MTH 165: Linear Algebra with Differential Equations

Midterm 1

October 13, 2016

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Indicate your instructor with a check in the box:

Bobkova	MWF 10:25-11:15	
Lubkin	MWF 9:00-9:50	
Rice	TR 14:00-15:15	
Vidaurre	MW 14:00-15:15	

- You have 75 minutes to work on this exam.
- No calculators, cell phones, other electronic devices, books, or notes are allowed during this exam.
- Show all your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You are responsible for checking that this exam has all 7 pages.

QUESTION	VALUE	SCORE
1	15	
2	15	
3	20	
4	20	
5	15	
6	15	
TOTAL	100	

1. (15 points) Solve the following initial value problem in explicit form.

$$e^{-x}\frac{dy}{dx} = \frac{6x^2e^{x^3} + 2e^{x^3}}{y}, \quad y(0) = -2.$$

2. (15 points) Find the general solution to the following differential equation.

$$(t^2 + 1)y' + 6ty = 30t(t^2 + 1)^2.$$

3. (20 points) Suppose a tank with a 40L capacity is initially filled with 10L of water in which 50g of salt is dissolved. A 3g/L solution is poured into the tank at a rate of 2L/min, while well-mixed solution is drained from the tank at a rate of 1L/min.

(a) How long does it take for the concentration of the solution in the tank to reach 4g/L?

(b) What is the concentration of the solution in the tank at the moment the tank begins to overflow?

4. (20 points) Consider the following system of equations, where x, y, z are the variables and k is a real constant.

$$x + 4y + 5z = 1$$

$$3x - y + z = 4$$

$$13y + kz = 2$$

(a) Determine which values of k cause the system to have one solution, no solutions, and infinitely many solutions, respectively.

(b) Solve the system with $k = 17$.

5. (15 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 2 & 5 & 7 \end{bmatrix}$$

(a) Find A^{-1}, or conclude that it does not exist.

(b) Find the matrix B that satisfies

$$BA - \begin{bmatrix} 1 & -1 & 2 \\ 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = 2\begin{bmatrix} 0 & 0 & -2 \\ 1 & -1 & -3 \\ -1 & 0 & -1 \end{bmatrix}$$

6. (15 points) Let $A = \begin{bmatrix} 3 & 7 & 1 \\ 0 & 5 & 2 \\ 3 & k & 5 \end{bmatrix}$, where k is a real number.

(a) Compute $\det(A)$ in terms of k.

(b) Determine all possible values of $\operatorname{rank}(A)$, along with which values of k cause those values to occur.