Math 165 Written Homework 9
Due Saturday, April 6th at 11:59 pm on gradescope

1. For each part below, determine if the given mapping T is linear or non-linear. If it is linear, prove it by verifying the necessary conditions. If it is not linear, provide an example showing that T fails to have the relevant property:

(a) $T : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ defined by $T(A) = AB - BA$, where B is a fixed $n \times n$ matrix.

(b) $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ defined by $T(A) = A^2$.

(c) $T : M_2(\mathbb{R}) \to \mathbb{R}$ defined by $T(A) = \det(A)$.

2. (a) Let $T_A : \mathbb{R}^4 \to \mathbb{R}^3$ be the matrix transformation corresponding to the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

This means $T_A(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathbb{R}^4$. Find a basis for the kernel of $T_{\mathbb{A}}$ and for the range of $T_{\mathbb{A}}$.

(b) Let $L : P_3 \to \mathbb{R}^2$ be the linear map $L(f) = (f(0), f(1))$. For example $L(x^2 + 1) = (1, 2)$. Find a basis for the kernel of L and for the range of L.