**Problems**

1. Prove that $W = \{p(x) \in P_1(\mathbb{R}) \mid 2p'(0) = p(1)\}$ is a subspace of $P_1(\mathbb{R})$. Determine a basis for $W$. Justify your answer. Determine $\dim(W)$.

2. Determine a basis for the following subspaces $W$ of the given vector spaces $V$. (You do not need to prove that $W$ is a subspace.)

   (a) $V = M_{3\times 3}(\mathbb{R})$. $W = \{A \in M_{3\times 3}(\mathbb{R}) \mid A^T = -A\}$.

   (b) $V = \mathbb{R}^4$, $W = \text{null}(A)$, where

   $$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 3 & -1 \\ 2 & -4 & -9 & 5 \end{bmatrix}$$

3. Determine whether the set $S = \{(1, 2, 0, -2), (1, 0, 3, 2), (0, -2, 1, 1)\} \subset \mathbb{R}^4$ is linearly independent. Then determine whether or not $\vec{b} = (1, 1, 1, 1)$ is in $\text{span}(S)$. Show your work to support your answer.

4. Let a set $S$ consist of 5 vectors in $\mathbb{R}^5$. Suppose we create a matrix $A$ such that the columns of $A$ are the transposes of the vectors of $S$.

   (a) Suppose $\text{rank}(A) = 5$. Is $S$ linearly independent? Does it span $\mathbb{R}^5$? Explain your answers, or explain why there is not enough information.

   (b) Suppose $\text{rank}(A) = 3$. Is $S$ linearly independent? Does it span $\mathbb{R}^5$? Explain your answers, or explain why there is not enough information.