Written Homework 6

Due, Friday, March 8 at midnight

1. Recall, that the dot product of an n-dimensional row vector $[a_1, \ldots, a_n]$ and an n-dimensional column vector $[b_1, \ldots, b_n]^T$ is given by $\sum_{k=1}^{n} a_k b_k$. We define the dot product of two n-dimensional column vectors $\vec{v}$ and $\vec{w}$ similarly as the matrix product $\vec{v}^T \vec{w}$ and similarly for two n-dimensional row vectors $\vec{a}, \vec{b}$ we define the dot product as $\vec{a}\vec{b}^T$. In all cases it comes down to the sum of products of corresponding entries of the vectors.
(a) In Euclidean geometry, the length of a vector $\vec{v}$ is denoted $||\vec{v}||$ and is given by the formula $||\vec{v}|| = \sqrt{\vec{v} \cdot \vec{v}}$ where $\cdot$ stands for dot product. Find the length of the vector $\vec{v} = (1, 2, 3)$.
(b) In Euclidean geometry, two n-dimensional vectors $\vec{v}, \vec{w}$ are said to be **orthogonal** if $\vec{v} \cdot \vec{w} = 0$. Given an $m \times n$ matrix $\mathbb{A}$, the **nullspace** of $\mathbb{A}$ is defined to be the set of solutions $\vec{x}$ to the homogeneous equation $\mathbb{A}\vec{x} = \vec{0}$. Explain why an n-dimensional column vector $\vec{x}$ lies in the nullspace of $\mathbb{A}$ if and only if it is orthogonal to all the rows of $\mathbb{A}$.
(c) Given the row vector $\vec{v} = (1, 2, 3)$, describe **all** the 3-dimensional column vectors $\vec{x}$ which are orthogonal to $\vec{v}$. (Hint: Consider $\mathbb{A}\vec{x} = \vec{0}$ where $\mathbb{A} = \vec{v}$.)
(d) Given the row vectors $\vec{v}_1 = (1, 1, 1)$ and $\vec{v}_2 = (1, 2, 3)$, describe **all** 3-dimensional column vectors $\vec{x}$ that are orthogonal to **both** $\vec{v}_1$ and $\vec{v}_2$.

2. (a) Let $V = \mathbb{R}^2$ be the vector space of 2-dimensional real row vectors. Decide which of the following subsets of V are **vector subspaces** of V and which aren't. Justify your answers.
(i) $L_1 = \{(x, y) | y = 3x\}$.
(ii) $L_2 = \{(x, y) | y = 3x + 1\}$.
(iii) $P = \{(x, y) | y = x^2\}$.
(b) Let $W = Mat_{2\times 2}(\mathbb{R})$ be the vector space of 2×2 real matrices. Decide which of the following subsets of W are **vector subspaces** of W and which

aren't. Justify your answers.
(i) $S = \{\mathbb{A} | tr(\mathbb{A}) = 0\}$ (Here tr stands for the trace of the matrix.)
(ii) $T = \{\mathbb{A} | tr(\mathbb{A}) = 1\}$
(iii) $R = \{\mathbb{A} | \det(\mathbb{A}) = 0\}$.