Written Homework 5

Due Friday, Mar 1 at 11:59 PM

1. Suppose $\mathbb{A}$ is a $n \times n$ matrix with determinant D.
(a) What is the determinant of $\mathbb{A}^k$, for any integer $k \geq 1$ as a function of D, k and n? (It is OK if it does not depend on all the variables stated.)
(b) What is the determinant of $\mathbb{B} = c\mathbb{A}$ for any real number c as a function of D, c and n?
(c) Suppose $\mathbb{A}$ has reduced row echelon form equal to the $n \times n$ identity matrix and in order to reduce it to this RREF you have to use r row swaps (the P_{ij} operations), and m operations of multiplying a row by a nonzero scalar where the scalars involved during the process were $\alpha_1, \alpha_2, \ldots, \alpha_m$. In addition you also use an unspecified number of operations of the type $A_{ij}(c)$ of adding a multiple of one row to a distinct row during the reduction process. Describe a formula for $D = \det(\mathbb{A})$ in terms of the variables $n, r, m, \alpha_1, \ldots, \alpha_m$ and explain how you got your answer.
(d) What can you say about $\det(\mathbb{A})$ if the RREF of $\mathbb{A}$ is not the $n \times n$ identity matrix?

2. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 0 & 2 & -6 \end{bmatrix}$.
(a) Find A^{-1}.
(b) Find $\det(A), \det(A^T)$, and $\det(A^{-1})$.
(c) Solve the system

$$\begin{aligned} x_1 + 2x_2 &= 1 \\ 2x_1 + 4x_2 + x_3 &= 0 \\ 2x_2 - 6x_3 &= 3 \end{aligned}$$