Written Homework 4

Due Friday, February 23 at midnight

1.

(a) Determine all values of the constant k for which the following system has (i) no solution, (ii) an infinite number of solutions, and (iii) a unique solution.

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 3 \\ 2x_1 + 5x_2 + x_3 &= 7 \\ x_1 + x_2 - k^2 x_3 &= -k \end{aligned}$$

In (ii) and (iii), provide a solution set in terms of k and an appropriate number of free parameters.

(b) Determine the solution set, if it exists, of the homogeneous system whose coefficient matrix is

$$A = \begin{bmatrix} 1-i & 2i \\ 1+i & -2 \end{bmatrix}.$$

2.

Let $A = \begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & 1 & 4 & 1 \\ -2 & 0 & -1 & 0 \\ -3 & 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} -5 & -5 & 5 & 7 \\ 6 & -1 & 13 & 20 \\ -3 & 0 & -11 & -19 \\ -1 & -2 & -5 & -11 \end{bmatrix}$.

Suppose a matrix B satisfies $AB = C$. Without inverting A, find the third column of B.