Homework 3 due Friday, February 16 at 11:59 pm on gradescope. The following homework submission standards will apply. If you do not meet them, the homework will be rejected and will count as one of the three that are dropped.

You must upload the pages in order. You must mark all the pages for each problem. You must match each all the problems (problem 1, problem 1.1, etc...) The pages must be rotated to the correct orientation. The homework should be legible The handwriting should be legible Don't try to squeeze things in with extra small writing The scan should be a decent quality (no shadows, etc...)

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1. In the complex numbers, we know that $i^2 = -1$. The goal of this exercise is to find all 2×2 matrices with real entries, that satisfy a similar equation. The following matrix plays the role analogous to -1:

$$-I = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Suppose

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and a, b, c, d are real numbers.

(a) Calculate M^2. (Your answer should give explicit expressions for each of the entries of M^2 in terms of a, b, c, d.)

(b) If $a + d \neq 0$ and $M^2 = -I$, explain why we must have $b = c = 0$ and use this to explain why there can be no such matrices with real entries.

(c) If $a + d = 0$ and $M^2 = -I$, give formulas for the entries of M purely in terms of the parameters a and b (do not use c or d). There is one value of b which will not occur in such matrices - what value is it?

(d) Give an example of a 2×2 real matrix M with $M^2 = -I$ where all entries are given as explicit numerical values (no parameters used).

2. Suppose that

$$A^{\#} = \begin{bmatrix} a & 1 & | & 2 \\ 1 & b & | & c \end{bmatrix}$$

is the augmented matrix of a linear system of equations in the variables x, y.
(a) Write out the system of two linear equations represented by the augmented matrix.
(b) Using the geometry of the xy-plane, find general conditions on the parameters a, b, and c so that this system has

(i) Exactly one solution,

(ii) no solutions, or

(iii) infinitely-many solutions.