

Math 165 - Spring 2024

Workshop 12

Due April 26th

Non-homogenous linear differential equations and Oscillation mechanical systems

Problem 1. What is the trial solution for each non-homogenous linear differential equation?

(a) $y'' - 2y' + 2y = \sin t + t \sin t + e^t \sin t$

(b) $y^{(5)} + y'' = 1 + e^t$

(c) $D^2(D^2 - 4I)(D^2 - 3D + 2I)y = 4te^{2t} + 1$

Problem 2. (a) Find the general solution for the following differential equation.

$$y^{(3)} + y' = e^t + \cos t$$

(b) What is the solution to the IVP problem, if we assume

$$\begin{cases} y(0) = 0 \\ y'(0) = 1 \\ y''(0) = 1 \end{cases} \quad ?$$

Problem 3. Consider the spring-mass system whose motion is governed by the differential equation

$$\frac{d^2y}{dt^2} + 2\alpha \frac{dy}{dt} + y = 0$$

Determine all values of the (positive) constant α for which the system is (i) underdamped, (ii) critically damped, and (iii) overdamped. In the case of overdamping, solve the system fully. If the initial velocity of the system is zero, determine if the mass passes through equilibrium.

Problem 4. Consider the spring-mass system whose motion is governed by the differential equation

$$y'' + 2y' + 5y = 17 \sin 2t$$

and satisfies $y(0) = -2$ and $y'(0) = 0$.

- (a) Determine whether the motion is underdamped, overdamped or critically damped.
(b) Find the solution to given initial-value problem.