Math 165 - Spring 2024
Workshop 11
Due April 19th
Eigenvalues and homogenous linear differential equation

Problem 1. Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(a) Find the eigenvalues and the corresponding eignespaces for A.
(b) Is A defective? If not, find the span of its eigenvectors?

Problem 2. Find the general solution of the following linear differential equations:
(a) $x^2 y'' + xy' - 3y = 0$ on $x > 0$.
(Hint: Use a trial solution of the form $y(x) = x^r$. Then identify what r must be.)
(b) $y'' - 3y' + 2y = 0$.
(c) $y^{(4)} - 4y^{(2)} = 0$.
(d) $y^{(4)} + 2y'' + y = 0$.

Problem 3. Solve the following IVP:

$$\begin{cases} y'' - 2y' + y = 0 \\ y(0) = 1 \\ y'(0) = -1 \end{cases}.$$