MTH 165 Written Homework 1
Due Friday, January 26, 2024 at 11:59PM on gradescope

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1. Consider the differential equation

$$\frac{dy}{dx} = \frac{y^2}{x^2+1}$$

1. Find the unique equilibrium solution for this differential equation.

2. Verify that the hypothesis of the existence and uniqueness theorem (Theorem 1.3.2 of the book) are satisfied.

3. For $y \neq 0$, use the method of separation of variables to solve the differential equation. Your solution should have an integration constant which you should call c. Where did you use the fact that $y \neq 0$?

4. Determine three particular solutions as follows: You'll see that $y(0) = \frac{-1}{c}$. Use geogebra or desmos to graph an example where $c \geq \frac{\pi}{2}$, one where $c \leq -\frac{\pi}{2}$, and one where$-\frac{\pi}{2} < c < \frac{\pi}{2}$. Sketch these solutions on the same coordinate axes. Now return to the slope field plotter to compare your graphs to solution curves in the slope field plotter. This last instruction is just for your understanding, not to turn in. (Go to geogebra.org and enter 'slope field plotter' in the search bar to get to the slope field plotter.)

2. Consider the initial value problem:

$$y' = y^2 + y - 2, \quad y(x_0) = y_0 \tag{1}$$

This is a first order ordinary differential equation with dependent variable y and independent variable x.

1. Verify that the hypotheses of the existence and uniqueness theorem (Theorem 1.3.2 of the book) are satisfied.

2. Determine all equilibrium solutions to this differential equation, i.e., all solutions where y is a constant.

3. Using your answers to the last two questions, explain why any solution $y = y(x)$ to this differential equation with $-2 < y_0 = y(x_0) < 1$ must have $-2 < y(x) < 1$ for all x and have y be a decreasing function of x.

4. Sketch a slope field for (1). Sketch representative curves for the three conditions $y_0 < -2$, $-2 < y_0 < 1$, and $y_0 > 1$. Use geogebra to check your sketches.

5. For a solution $y = y(x)$ to this differential equation with $y_0 = y(x_0) = 0$, what are $\lim_{x\to\infty} y(x)$ and $\lim_{x\to-\infty} y(x)$ equal to?