MATH 165: WRITTEN HW 6

DUE: FRIDAY, OCT 25, 11:59PM ON GRADESCOPE
UNIVERSITY OF ROCHESTER, FALL 2024

Problem 1. The **dot product** of an n-dimensional row vector $\begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}$ and an n-dimensional column vector $\begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix}^T$ is given by

$$\sum_{k=1}^{n} a_k b_k = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n.$$

In general, we define the dot product of two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^n$ (so each of them can be any of n-tuple, row vector, or column vector) similarly as the sum of products of corresponding entries of the vectors.

(a) In Euclidean geometry, the **length** of a vector $\vec{v}$ in $\mathbb{R}^n$ is denoted by $\|\vec{v}\|$ and given by the formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

where $\cdot$ stands for dot product. Find the length of the vector $\vec{v} = (1, 2, 3)$.

(b) Two vectors $\vec{v}, \vec{w}$ in $\mathbb{R}^n$ are said to be **orthogonal** if $\vec{v} \cdot \vec{w} = 0$. Explain why an n-dimensional column vector $\vec{x}$ lies in the null space of an $m \times n$ matrix A if and only if it is orthogonal to **all** the row vectors of A.

(c) Given the row vector $\vec{v} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, describe all the 3-dimensional column vectors $\vec{x}$ which are orthogonal to $\vec{v}$. (*Hint*: Consider $A\vec{x} = 0$ where $A = \vec{v}$.)

(d) Given the row vectors $\vec{v_1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ and $\vec{v_2} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, describe all 3-dimensional column vectors $\vec{x}$ that are orthogonal to **both** $\vec{v_1}$ and $\vec{v_2}$.

Problem 2. (a) Let $V = \mathbb{R}^2$. Decide which of the following subsets of V are **subspaces** of V and which are not. Justify your answers.

- (i) $L_1 = \{(x, y) : y = 3x\}$.
- (ii) $L_2 = \{(x, y) : y = 3x + 1\}$.
- (iii) $P = \{(x, y) : y = x^2\}$.

(b) Let $W = M_2(\mathbb{R})$ be the vector space of 2×2 real matrices. Decide which of the following subsets of W are **subspaces** of W and which are not. Justify your answers.

- (i) $S = \{A : \mathrm{tr}(A) = 0\}$ (here tr stands for the trace of the matrix).
- (ii) $T = \{A : \mathrm{tr}(A) = 1\}$.
- (iii) $R = \{A : \det(A) = 0\}$.