

MATH 165: WRITTEN HW 5

DUE: FRIDAY, OCT 18, 11:59PM ON GRADESCOPE
UNIVERSITY OF ROCHESTER, FALL 2024

Problem 1. Let A be an $n \times n$ matrix with determinant D .

- (a) What is the determinant of A^k , for any integer $k \geq 1$ as a function of D, k and n ? (*Note.* It may not depend on all the variables stated.)
- (b) What is the determinant of $B = cA$ for any real number c as a function of D, c and n ?
- (c) Suppose that the RREF (reduced row-echelon form) of A is the $n \times n$ identity matrix. Furthermore, suppose that in order to reduce it to this RREF, one used r row swaps (Type I operations), m operations of multiplying a row by a nonzero scalar (Type II operations) where the scalars involved during the process were $\alpha_1, \alpha_2, \dots, \alpha_m$, and an unspecified number of operations of adding a multiple of one row to a distinct row (Type III operations) during the reduction process. Describe a formula for $D = \det(A)$ in terms of the variables $n, r, m, \alpha_1, \dots, \alpha_m$ and explain how you got your answer.
- (d) What can you say about $\det(A)$ if the RREF of A is not the $n \times n$ identity matrix?

Problem 2. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 0 & 2 & -6 \end{bmatrix}$.

- (a) Find A^{-1} by reducing the 3×6 matrix $[A \mid I]$ to its RREF.
- (b) Find $\det(A)$, $\det(A^T)$, and $\det(A^{-1})$.
- (c) Solve the following system of linear equations.

$$\begin{aligned}x_1 + 2x_2 &= 1 \\2x_1 + 4x_2 + x_3 &= 0 \\2x_2 - 6x_3 &= 3\end{aligned}$$