MATH 165: WRITTEN HW 4

DUE: FRIDAY, OCT 4, 11:59PM ON GRADESCOPE
UNIVERSITY OF ROCHESTER, FALL 2024

Problem 1. Determine all values of the constant k for which the following system has

(i) no solution,

(ii) a unique solution, and

(iii) infinitely many solutions.

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 3 \\ 2x_1 + 5x_2 + x_3 &= 7 \\ x_1 + x_2 - k^2 x_3 &= -k \end{aligned}$$

In (ii) and (iii), provide a solution set in terms of k and an appropriate number of free variables.

Problem 2. Let

$$A = \begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & 1 & 4 & 1 \\ -2 & 0 & -1 & 0 \\ -3 & 1 & -1 & -2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & -3 & 1 & 0 \\ 1 & -1 & 2 & -5 \\ -1 & 0 & -2 & 2 \\ -1 & -2 & -3 & 3 \end{bmatrix}.$$

Suppose a matrix B satisfies $AB = C$.

(a) Denote the first column of B by $\vec{v_1}$. Explain why $\vec{v_1}$ should satisfy the vector equation

$$A\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix},$$

and determine $\vec{v_1}$ by solving the corresponding system of linear equations using **Gauss-Jordan elimination**.

(b) Denote the second column of B by $\vec{v_2}$. What should be $A\vec{v_2}$? Determine $\vec{v_2}$ by solving the corresponding system of linear equations, again using Gauss-Jordan elimination.

(c) Reduce the 4×8 matrix $\left[A \mid C\right]$ to its **reduced row-echelon form (RREF)**. Explain why the right part of the RREF should be equal to B. (*Remark. You should not invert A to find B directly.*)