MATH 165: WRITTEN HW 3

Problem 1. In the complex numbers, we know that $i^2 = -1$. The goal of this exercise is to find a 2×2 matrix with real entries, that satisfies a similar equation. The following matrix plays the role analogous to -1:

$$-I = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Suppose

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

and a, b, c, d are real numbers.

(a) Calculate M^2. (Your answer should give explicit expressions for each of the entries of M^2 in terms of a, b, c, d.)

(b) If $a + d \neq 0$ and $M^2 = -I$, explain why we must have $b = c = 0$ and use this to explain why there can be no such matrices with real entries.

(c) If $a + d = 0$ and $M^2 = -I$, give formulas for the entries of M purely in terms of the parameters a and b (that is, represent c and d in terms of a and b). There is one value of b which will not occur in such matrices - what value is it?

(d) Give an example of a 2×2 real matrix M with $M^2 = -I$ where all entries are given as explicit numerical values (no parameters used).

Problem 2. Suppose that

$$A^{\#} = \left[\begin{array}{cc|c} a & 1 & 2 \\ 1 & b & c \end{array}\right]$$

is the augmented matrix of a system of linear equations in the variables x, y.

(a) Write out the system of two linear equations represented by the augmented matrix.

(b) Using the geometry of the xy-plane, find general conditions on the parameters a, b, and c so that this system has

    (i) exactly one solution,

    (ii) no solution, or

    (iii) infinitely many solutions.