MATH 165: WRITTEN HW 1

DUE: FRIDAY, SEP 13, 11:59PM ON GRADESCOPE
UNIVERSITY OF ROCHESTER, FALL 2024

Problem 1. Consider the differential equation

$$\frac{dy}{dx} = \frac{y^2}{x^2+1}.$$

(a) Verify that the hypotheses of the Existence and Uniqueness Theorem (Theorem 1.3.2 of the book) are satisfied.

(b) Determine all equilibrium solutions to the given differential equation.

(c) For $y \neq 0$, use the method of separation of variables to solve the differential equation. Your solution should have an integration constant which you should call C. Where did you use the fact that $y \neq 0$?

(d) Explain why all solutions to this differential equation with $y \neq 0$ are increasing functions of x.

(e) Sketch a solution with $y(0) < 0$. Use the slope field plotter linked on the homework page to check your sketch. For such a solution, what is $\lim_{x\to\infty} y(x)$?

Problem 2. Consider the initial value problem:

$$y' = y^2 + y - 2, \quad y(0) = y_0$$

Here y_0 is a given real number.

(a) Verify that the hypotheses of the Existence and Uniqueness Theorem (Theorem 1.3.2 of the book) are satisfied.

(b) Determine all equilibrium solutions to the given differential equation.

(c) Sketch a slope field for the given differential equation. Sketch representative solution curves for the three conditions $y_0 < -2$, $-2 < y_0 < 1$, and $y_0 > 1$. (For example, you may sketch three solution curves, passing through $(0, -3)$, $(0, 0)$, and $(0, 2)$, respectively.) Use the slope field plotter linked on the homework page to check your sketches.

(d) Suppose that $-2 < y_0 < 1$ and $y = y(x)$ is a particular solution to the given initial value problem. Using your answers to the last two questions, explain why $-2 < y(x) < 1$ for all x and $y(x)$ is a decreasing function of x.

(e) Suppose that $y_0 = 0$ and $y = y(x)$ is a particular solution to the given initial value problem. What are $\lim_{x\to\infty} y(x)$ and $\lim_{x\to-\infty} y(x)$ equal to?