MTH 165

Midterm 2

04/04/2020

Name: ______________________________

UR ID: ______________________________

Circle your Instructor's Name:

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- **Show work and justify all answers. You may not receive full credit for a correct answer if insufficient work or justification is given. Please record your final answers in the answer boxes.**
- **You are responsible for checking that this exam has all 6 problems.**

HONOR PLEDGE:

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

YOUR SIGNATURE:____________________________________

1. (10 points) Let A, B and $C \in M_{n\times n}(\mathbb{R})$ be such that C is an invertible matrix and $AB = C^3$. Show that B is an invertible matrix.

2. (10 points) Given that

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 1,$$

find the following determinant:

$$\det \begin{bmatrix} 2a+d & 2b+e & 2c+f \\ 2d+g & 2e+h & 2f+i \\ 2g+a & 2h+b & 2i+c \end{bmatrix}.$$

Completely justify your answer.

3. (20 points) For the following vector spaces V, determine whether or not the given set W is a subspace of V. In either case, justify your answer thoroughly.

(a) $V = \mathbb{R}^4$ and $W = \{(x_1, x_2, x_3, x_4) \in V \,|\, x_1 + x_2 = x_3 + x_4\}$;

(b) $V = P_4(\mathbb{R})$ (i.e., the vector space of polynomials with real coefficients, having degree at most equal to 4) and $W = \{p \in V \,|\, p(1)p(2) = 2p(3)\}$.

4. (10 points) Let

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \quad \text{and} \quad A_3 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}.$$

Determine whether $A \in \operatorname{span}\{A_1, A_2, A_3\}$ or not. If so, express A as a linear combination of A_1, A_2, and A_3. Otherwise, explain why this is not possible.

5. (10 points) Let $V = M_{3\times 2}(\mathbb{R})$ and consider the subspace

$$W = \{A \in V \,|\, a_{11} + a_{22} = 3a_{12} + 2a_{32}\}.$$

Find a spanning set for W. Completely justify your answer.

6. (10 points) Is the set of polynomials

$$\{x^3 - x, 2x^2 + 4, -2x^3 + 3x^2 + 2x + 6\} \subseteq P_3(\mathbb{R})$$

linearly independent? If so, justify why; if not, find an explicit linear dependence.