Midterm 1, MTH 165, Spring 2020

1. An object whose temperature is 615°F is placed in a room whose temperature is 75°F. At 4 p.m., the temperature of the object is 135°F, and an hour later its temperature is 95°F. It is assumed that the temperature of the object $T = T(t)$ varies according to Newton's law of cooling, i.e.,

$$\frac{dT}{dt} = -k\,(T - T_m),$$

where T_m is the temperature of the surrounding medium and k is a positive constant. Under these assumptions, find the time at which the object was placed in the room.

2. Find the general solution of the differential equation

$$x\,y'' - y' \;=\; 4x^3.$$

(Hint: You may find useful letting $v = y'$ and rewriting the equation solely in terms of v.)

3. A 450-gal tank initially contains 50 gal of pure water. Brine enters the tank through two faucets: one containing 0.2 lb/gal of salt flows in at the rate of 3 gal/min, while the second one containing 0.6 lb/gal of salt flows in at the rate of 5 gal/min. The well-stirred mixture flows out of the tank at the rate of 4 gal/min. How much salt is in the tank just before the solution overflows?

4. We recall that a square matrix C is symmetric if $C = C^T$, where C^T denotes its transpose. Using properties of the transpose and the above definition, show that AA^T is a symmetric matrix if A is an arbitrary, not necessarily square matrix.

5. Consider the matrix

$$A = \begin{bmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 4 & 4 & 8 & 0 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{bmatrix}$$

(a) By performing elementary row operations, determine its reduced row-echelon form. Indicate which row operations are used in each step.

(b) Find the rank of matrix A and explain your answer.

6. Use Gauss-Jordan elimination to solve the system

$$\begin{cases} 3x_1 + 2x_2 + 3x_3 - 2x_4 = 1, \\ x_1 + x_2 + x_3 = 3, \\ x_1 + 2x_2 + x_3 - x_4 = 2. \end{cases}$$