Midterm 1, MTH 165, Spring 2019

1. Find the equation of the curve that passes through the point $(0, 1/2)$ and whose slope at each point (x, y) is $-\frac{x}{4y}$.

2. Find the general solution of the differential equation

$$1 - y \sin x - \cos x \frac{dy}{dx} = 0.$$

3. A tank initially contains 10 L of a salt solution. Water flows into the tank at a rate of 3 L/min, and the well-stirred mixture flows out at a rate of 2 L/min. After 5 minutes, the concentration of salt in the tank is 0.2 g/L. Find:

(a) The amount of salt in the tank initially;

(b) The volume of the solution in the tank when the concentration of salt is 0.025 g/L.

4. Consider the matrix

$$A = \begin{bmatrix} 3 & -2 & -1 & 17 \\ 2 & 2 & -4 & 8 \\ -1 & 4 & -3 & 1 \end{bmatrix}$$

(a) By performing elementary row operations, determine its reduced row-echelon form. Indicate which row operations are used in each step.

(b) Find the rank of matrix A and explain your answer.

5. Consider the system

$$\begin{cases} 2x_1 + x_2 - x_3 + x_4 = 0, \\ x_1 + x_2 + x_3 - x_4 = 0, \\ 4x_1 + 2x_2 - x_3 + x_4 = 0, \\ 3x_1 - x_2 + x_3 + cx_4 = 0. \end{cases}$$

(a) Find all values of the constant c for which the system is consistent.

(b) For the case when the system is consistent, use Gaussian (or Gauss-Jordan) elimination to determine the solution set.

6. Consider the differential equation

$$y' = y^2(y - 1).$$

(a) Find all of its equilibrium solutions.

(b) Determine the regions in the xy-plane where the solutions are increasing.

(c) Determine the regions in the xy-plane where the solution curves are concave up.