MTH 165: Linear Algebra with Differential Equations

Final Exam

December 17, 2016

NAME (please print legibly): ____
Your University ID Number: ____
Indicate your instructor with a check in the box:

Bobkova	MWF 10:25-11:15	
Lubkin	MWF 9:00-9:50	
Rice	TR 14:00-15:15	
Vidaurre	MW 14:00-15:15	

- You have 180 minutes to work on this exam.
- No calculators, cell phones, other electronic devices, books, or notes are allowed during this exam.
- **Show all your work and justify your answers, even if the answer is just a number or "yes/no" or "true/false".** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You are responsible for checking that this exam has all 13 pages.

QUESTION	VALUE	SCORE
1	20	
2	36	
3	24	
4	36	
5	36	
6	24	
7	24	
TOTAL	200	

1. (20 points) Solve the following initial value problems in explicit form.

(a) $y' = 4e^{2x-y}$, $y(0) = 0$

(b) $y' = 4e^{2x} - y$, $y(0) = 0$

2. (36 points) Determine if the following statements are true or false.

(a) The polynomials $p_1(x) = 1$, $p_2(x) = x^2 + x$, $p_3(x) = 2 + 5x$, and $p_4(x) = x^2 - 1$ are linearly independent vectors in $P_2(\mathbb{R})$, the space of polynomials of degree at most 2.

(b) The vectors $\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$ form a basis for $\mathbb{R}^3$.

(c) The set $S = \{(x, y) : x + y \geq 0\}$ is a subspace of $\mathbb{R}^2$.

(d) The transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ defined by $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} a+b \\ c+d \\ ab \\ cd \end{bmatrix}$ is linear.

3. (24 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 3 & -5 \\ 0 & 2 & -3 \end{bmatrix}.$$

(a) Determine all eigenvalues of A.

(b) If A^{-1} exists, compute it. If A^{-1} does not exist, explain why not.

4. (36 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 5 & 7 \\ 3 & 6 & 9 & 16 & 22 \\ 5 & 10 & 15 & 26 & 36 \end{bmatrix},$$

and suppose the function $T : \mathbb{R}^5 \to \mathbb{R}^3$ is defined by $T(\mathbf{x}) = A\mathbf{x}$.

(a) Determine a row-echelon form of A.

(b) Determine a basis for the range (also known as the image) of T.

(c) What is the dimension of the kernel of T?

(d) Are the rows of A linearly independent?

5. (36 points)

(a) Find the general real-valued solution to the differential equation

$$y^{(4)} - 25y'' = 0.$$

(b) Find a particular real-valued solution to the differential equation

$$y'' + 9y = 30e^{x}.$$

(c) Find a particular real-valued solution to the differential equation

$$y'' + 9y = 12\sin(3x).$$

(d) Solve the initial value problem

$$y'' + 9y = 30e^{x} + 12\sin(3x), \quad y(0) = 4,\ y'(0) = 3.$$

6. (24 points) Consider a spring-mass system with spring constant $k = 4$ N/m, and a mass $m = 1$ kg, which is hung from the spring and allowed to reach equilibrium. Determine the following about the position function y of the mass, relative to equilibrium, where the positive direction is downward.

(a) If the medium has damping constant $c = 4$, no external force is acting on the system, and the mass is stretched 1 m past equilibrium and released, find a formula for y.

(b) If there is no damping force, and an external force of $F(t) = \sin(t)$ acts on the system, describe all possible formulas (i.e. give the *general real-valued solution*) for y.

7. (24 points)

(a) Solve the following initial value problem.

$$x' = 2x + y$$

$$y' = 2x + 3y$$

$$x(0) = 1,\ y(0) = 2.$$

(b) Find the general, real-valued solution to the following system of equations.

$$x' = 3x + 2y$$

$$y' = -x + y.$$