Math 164: Multidimensional Calculus

Midterm II

November 17, 2016

NAME (please print legibly):
Your University ID Number:
Indicate your instructor with a check in the appropriate box:

Kleene	TR 12:30-1:45pm	
Salur	MW 3:25-4:40pm	
Gafni	TR 3:25-4:40pm	
Lee	MWF 09:00-09:50am	

- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 10 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers.
- Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

QUESTION	VALUE	SCORE
1	10	
2	15	
3	10	
4	15	
5	25	
6	15	
7	10	
TOTAL	100	

1. (10 points) Consider the function

$$f(x, y, z) = \frac{2x^2}{x+z} + x^2y^2e^z - xy\sin(z) + \frac{e^{\sqrt{x+y}}}{\cos(xy)}.$$

Evaluate the third order partial derivative $f_{xyz}(1, 1, 0)$.

2. (15 points) Let $f(x, y)$ be given by

$$f(x, y) = 4 - \ln(xy - 9) + x^2 cos(\pi y).$$

a) Explain why $f(x, y)$ is differentiable at $(5, 2)$.

b) Find an equation for the linearization of f at the point $(5, 2)$, and use it to estimate the value of f at the point $(5.05, 1.98)$.

3. (10 points) Write an equation for the tangent plane to the surface $x^2 + 2y^2 + 3z^2 = 6$ at the point $(1, 1, 1)$.

4. (15 points) Find the directional derivative of $f(x, y) = 2x^2y^3 + 6xy$ at (1,1) in the direction of a unit vector whose angle with the positive x-axis is $\pi/6$. (Recall that $\cos(\pi/6) = \sqrt{3}/2$)

5. (25 points) **a)** Consider the function $f(x, y) = x^2y - 2xy + 2y^2 - 15y$. Find all the critical points of f and classify them as relative maxima, relative minima, or saddle points.

b) Find the extreme values of the function $f(x, y) = 3x^2 + 2y^2 - 4y$ subject to the constraint $x^2 + y^2 \leq 9$. Let $D = \{(x,y) \mid x^2+y^2 \leq 9\}$. D is closed and bdd so extrema are attained.

On the boundary, use Lagrange multipliers ($g(x,y) = x^2 + y^2 = 9$):

$$\left.\begin{aligned}\nabla f &= \langle 6x, 4y-4\rangle \\ \nabla g &= \langle 2x, 2y\rangle\end{aligned}\right\} \Rightarrow \nabla f = \lambda \nabla g \text{ gives } \begin{aligned} 6x &= \lambda 2x \\ 4y-4 &= \lambda 2y\end{aligned}$$

① $6x = \lambda 2x \Rightarrow 2x(3-\lambda) = 0 \Rightarrow x = 0$ or $\lambda = 3$

② $4y - 4 = \lambda 2y \Rightarrow 4y - 2\lambda y - 4 = 0$

③ $x^2 + y^2 = 9$

① and ② $\Rightarrow$ if $\lambda = 3$, $4y - 6y - 4 = 0 \Rightarrow -2y = 4 \Rightarrow y = -2 \overset{\text{by ③}}{\Rightarrow} x = \pm\sqrt{5} \Rightarrow$

$(\sqrt{5}, -2) \rightarrow f = 15 - 4 + 8 = 19$

$(-\sqrt{5}, -2) \rightarrow f = 19$

① and ③ $\Rightarrow$ if $x = 0$, $y^2 = 9 \Rightarrow y = \pm 3 \Rightarrow$

$(0,3) \rightarrow f = 0 + 18 - 12 = 6$

$(0,-3) \rightarrow f = 0 + 18 + 12 = 30$

On the interior, check for critical points: $f_x = 0$ when $x = 0$

$f_y = 0$ when $4y = 4$, or $y = 1$

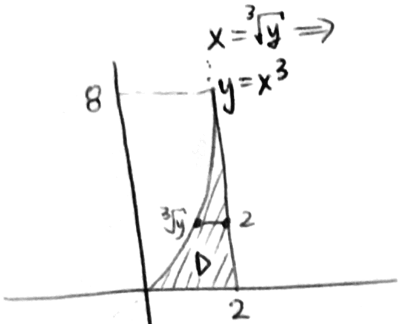
$(0,1) \rightarrow f = 0 + 2 - 4 = -2$

So, we must compare the values $-2, 6, 19$, and 30

The abs. max. of f on D is 30, attained at $(0,-3)$.

The abs. min. of f on D is -2, attained at $(0,1)$.

6. (15 points) Evaluate the integral by reversing the order of integration.



$$I = \int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy = \int_0^2 \int_0^{x^3} e^{x^4}\, dy\, dx$$

$D = \{(x,y) \mid 0 \le y \le 8,\ \sqrt[3]{y} \le x \le 2\}$

$D = \{(x,y) \mid 0 \le 2 \le x,\ 0 \le y \le x^3\}$

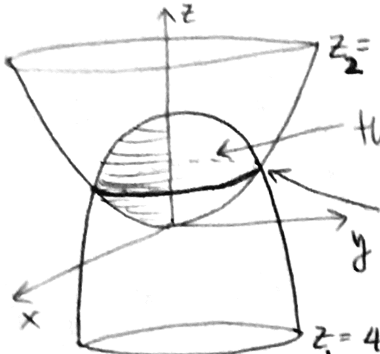
$$I = \int_0^2 \left[y e^{x^4} \right]_0^{x^3} dx = \int_0^2 x^3 e^{x^4}\, dx = \left[\frac{1}{4} e^{x^4} \right]_0^2 = \frac{1}{4}(e^{16} - e^0)$$

$u = x^4$

$\frac{1}{4} du = x^3 dx$

$$= \boxed{\frac{1}{4}(e^{16} - 1)}$$

7. (10 points) Use polar coordinates to find the volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.



$3x^2+3y^2 = 4-x^2-y^2$

or $4x^2+4y^2 = 4$

or $x^2+y^2 = 1$

or $r=1$

so $D = \{(x,y) \mid x^2+y^2 \le 1\}$

or $D = \{(r,\theta) \mid 0 \le r^2 \le 1,\ 0 \le \theta \le 2\pi\}$
$= \{(r,\theta) \mid 0 \le r \le 1,\ 0 \le \theta \le 2\pi\}$

$$\text{Volume} = \iint_D (z_1 - z_2)\, dA$$

$$= \int_0^{2\pi}\int_0^1 ((4-r) - 3r^2)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 (4r - r^2 - 3r^3)\, dr\, d\theta = \int_0^{2\pi} \left[2r^2 - \tfrac{1}{3}r^3 - \tfrac{3}{4}r^4\right]_0^1 d\theta$$

$$= \left(2 - \tfrac{1}{3} - \tfrac{3}{4}\right)2\pi = \boxed{\tfrac{26}{12}\pi}$$