Math 164 Exam 2 Formulas

1. Implicit Differentiation for $z = f(x, y)$:

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

2. Gradient of $f(x, y)$:

$$\nabla f(x, y) = \langle f_x, f_y \rangle$$

Gradient of $f(x, y, z)$:

$$\nabla f(x, y, z) = \langle f_x, f_y, f_z \rangle$$

3. The tangent plane to the surface $z = f(x, y)$ at the point (x_0, y_0, z_0) is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

4. The tangent plane to the level surface $F(x, y, z) = k$ at the point (x_0, y_0, z_0) is

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0.$$

5. The linearization of f at (a, b) is

$$L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

6. The directional derivative of f in the direction of $\mathbf{u}$:

$$D_{\mathbf{u}} f = \nabla f \cdot \mathbf{u}$$

7. The general formula for the derivative of $\ln(u)$ is:

$$\frac{d}{dx} \ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$$

For example if $f(x, y) = \ln(x + y^2)$, then

$$f_x(x, y) = \frac{\partial}{\partial x} \ln(x + y^2) = \frac{1}{x + y^2}$$

and

$$f_y(x, y) = \frac{\partial}{\partial y} \ln(x + y^2) = \frac{1}{x + y^2} \cdot \frac{\partial}{\partial y}(x + y^2) = \frac{2y}{x + y^2}$$