Formulas

Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance from a point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Scalar projection of $\mathbf{v}$ onto $\mathbf{u}$:

$$\text{comp}_{\mathbf{u}}(\mathbf{v}) = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|}$$

Vector projection of $\mathbf{v}$ onto $\mathbf{u}$:

$$\text{proj}_{\mathbf{u}}(\mathbf{v}) = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|^2}\mathbf{u}$$

Vector equation of a line:

$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$$

Vector equation of a plane:

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

Arc Length Formula from $t = a$ to $t = b$:

$$L = \int_a^b |\mathbf{r}'(t)|\, dt$$

Arc Length Function for $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ starting at $t = a$:

$$s(t) = \int_a^t |\mathbf{r}'(u)|\, du = \int_a^t \sqrt{\left(\frac{df}{du}\right)^2 + \left(\frac{dg}{du}\right)^2 + \left(\frac{dh}{du}\right)^2}\, du$$

Unit tangent vector for $\mathbf{r}(t)$:

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

Implicit Differentiation for $F(x, y, z(x, y)) = 0$:

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad\qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Gradient of $f(x, y, z)$:

$$\nabla f(x, y, z) = \langle f_x, f_y, f_z \rangle$$

The directional derivative of f in the direction of $\mathbf{u}$:

$$D_{\mathbf{u}} f = \nabla f \cdot \mathbf{u}$$

The tangent plane to the surface $z = f(x, y)$ at the point (x_0, y_0, z_0) is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$