Math 164: Multivariable Calculus

Midterm Exam 2

November 15, 2012

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

CIRCLE YOUR INSTRUCTOR: **Fili** **Madhu** **Mahmood**

- **NO calculators, cell phones, iPods or other electronic devices are allowed during the exam.**
- **Show your work and justify your answers. You may not receive credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **There is no need to simplify your answers.**

QUESTION	VALUE	SCORE
1	15	
2	15	
3	15	
4	10	
5	15	
6	15	
7	15	
TOTAL	100	

1. (15 points)

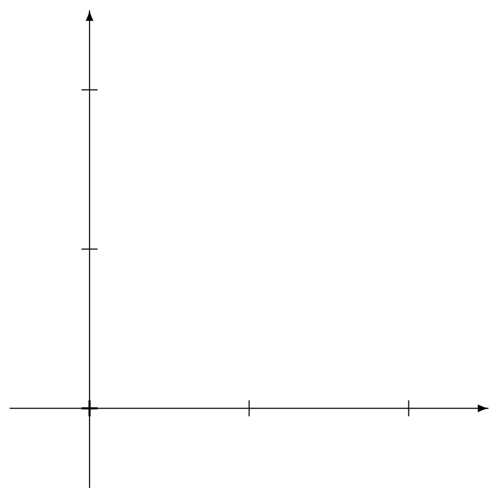
Find the extremum points and the maximum and minimum values of the function $f(x, y) = e^{-xy}$ subject to the constraint $x^2 - xy + y^2 = 9$.

2. (15 points)

Consider the double integral

$$\int_0^{\pi/4} \int_0^{1/\cos\theta} r^3 \, dr \, d\theta.$$

(a) Sketch the region of integration.



(b) Convert the above integral to rectangular coordinates.

(c) Change the order of integration in rectangular coordinates.

(d) Evaluate the integral in any form you desire.

3. (15 points)

Consider a lamina that occupies the region in the xy-plane bound by $x = 0$, $y = 0$ and the parabola $y = 1 - x^2$. The lamina has density function $\rho(x, y) = x + 1$.

(a) Find the mass of the lamina.

(b) Find the x-coordinate of the center of mass of the lamina.

4. (10 points)

Let B be the ball $x^2 + y^2 + z^2 \leq a^2$ of radius a. Its volume $\dfrac{4\pi a^3}{3}$ can be computed as a triple integral $\displaystyle\iiint_B dV$. Set up iterated integrals for computing this triple integral in

(a) rectangular coordinates.

(b) cylindrical coordinates.

(c) spherical coordinates.

5. (15 points)

Evaluate the triple integral by converting to cylindrical coordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_1^{2-\sqrt{x^2+y^2}} \frac{xy}{(x^2+y^2)\sqrt{x^2+y^2}}\, dz\, dy\, dx$$

6. (15 points)

Let R be the triangular region in the xy-plane whose vertices have coordinates $(-1,-3),(3,1)$ and $(0,0)$. Let T be the transformation from the uv-plane to the xy-plane given by

$$T(u,v) = (3u + v, 3v + u).$$

(a) There are three points in the uv-plane whose images under the transformation are the vertices of the triangle. What are these three points?

(b) What is the Jacobian of this transformation?

(c) Evaluate the following double integral:

$$\iint_R (x - 3y)\, dA$$

7. (15 points)

Evaluate

$$\iiint\limits_{\mathcal{B}} z e^{(x^2+y^2+z^2)^2}\, dV$$

where $\mathcal{B}$ is the hemispherical solid bounded by the sphere of radius 2 centered at the origin and the xy-plane, with $z \geq 0$.