Math 164: Multivariable Calculus

Final Exam

December 16, 2012

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

CIRCLE YOUR INSTRUCTOR: **Fili** **Madhu** **Mahmood**

- **NO calculators, cell phones, iPods or other electronic devices are allowed during the exam.**
- **Show your work and justify your answers. You may not receive credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **There is no need to simplify your answers.**

Part A		
QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
6	20	
7	20	
8	20	
9	20	
10	20	
TOTAL	100	

Part A

1. (20 points)

(a) Find the equation of the tangent plane to the surface $x^2 + y^2 - z^2 = -1$ at the point $(1, 1, \sqrt{3})$.

(b) Suppose you head toward the xy-plane from the surface $x^2 + y^2 - z^2 = -1$ at the point $(1, 1, \sqrt{3})$ by following the normal line to the surface at that point.

1. What are the x and y-coordinates at which you will hit the xy-plane?

(continued on next page...)

2.How far do you travel before you reach the xy-plane along this line?

2. (20 points) Find the limit if it exists. If it does not exist, explain why.

$$\lim_{(x,y)\to(0,0)} \frac{y^2 \sin^2(x)}{x^4 + y^4}$$

3. (20 points) Find the maximum and minimum values of the function $f(x, y) = xy$ subject to the constraint $18x^2 + 2y^2 = 36$.

4. (20 points) Find and classify all critical points for the function $f(x, y) = 6xy^2 - 3x^2 + 2x^3 + 3y^2$.

5. (20 points) Let D be the elliptical region given by $\dfrac{x^2}{4} + \dfrac{y^2}{9} \leq 1$. Compute

$$\iint_D x^2 \, dx \, dy.$$

Hint: Start with the change of variables $x = 2u, y = 3v$.

Part B

6. (20 points) Let C be the curve satisfying $x^2 + y^2 = 4$, $y \geq 0$, with counterclockwise orientation.

(a) Find the **average value** of the scalar function $f(x, y) = 1 + yx^2$ on the curve C.

(b) Suppose that a particle moves along C in the presence of a force field $\mathbf{F}(x, y) = \langle 3x^2, y \rangle$. Compute the work $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(c) Is the field $\mathbf{F}(x, y) = \langle 3x^2, y \rangle$ from the previous question conservative? Why or why not?

(d) Suppose γ is a smooth oriented curve which starts at $(-1, 0)$ and ends at $(3, 4)$. Compute $\int_\gamma \mathbf{F} \cdot d\mathbf{r}$ with $\mathbf{F}$ as above.

7. (20 points) Suppose that D is a region in $\mathbb{R}^2$ with area 4. Compute

$$\oint_{\partial D} \left(\frac{1}{2}y^2 e^x - 2y\right) dx + (\sin y + ye^x)\, dy.$$

Be sure to *justify* your answer.

8. (20 points) Let S be the helicoid given by the parametrization $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, v\rangle$ with $0 \le u \le 1$ and $0 \le v \le \pi$. Compute

$$\iint_S \sqrt{x^2+y^2}\, dS.$$

9. (20 points) Consider the solid cube $E = \{(x, y, z) : 0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2\}$. The boundary surface S of E is a closed six-sided box. Define a vector field on $\mathbb{R}^3$ by $\mathbf{F}(x, y, z) = xz\mathbf{i} + 3xz\mathbf{j} + 2z^2\mathbf{k}$.

(a) Find $\operatorname{div} \mathbf{F}$.

(b) Find the flux of $\mathbf{F}$ through the top face of S (at $z = 2$), oriented upwards.

(c) Use the Divergence Theorem to find the flux of $\mathbf{F}$ through S.

(d) Find the flux of $\mathbf{F}$ through the five-sided open box obtained by removing the top face from S.

10. (20 points)

Let S be the hemisphere $x^2 + y^2 + z^2 = 1$, $x \geq 0$, oriented so that the normal vector always points away from the origin. Let $\mathbf{F}(x, y, z) = -y\mathbf{i} + 3x\mathbf{j} + e^{xy}\mathbf{k}$.

(a) Find curl $\mathbf{F}$.

(b) Give a parametrization for the boundary C of the surface S.

(c) Use Stokes' Theorem to calculate the surface integral $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.