Math 164: Multidimensional Calculus

Midterm 1

October 15, 2015

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the appropriate box:

Bobkova	TR 12:30-1:45pm	
Chen	MW 3:25-4:40pm	
Dummit	TR 3:25-4:40pm	
Salur	MWF 09:00-09:50am	

- You have 75 minutes to work on this exam.

- You are responsible for checking that this exam has all 8 pages.

- No calculators, phones, electronic devices, books, notes are allowed during the exam.

- Show all work and justify all answers.

- Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: _____

QUESTION	VALUE	SCORE
1	16	
2	15	
3	15	
4	12	
5	12	
6	15	
7	15	
TOTAL	100	

1. (16 points) Let $\mathbf{v} = \langle -4, 3, 0 \rangle$ and $\mathbf{w} = \langle 2, -1, 2 \rangle$. Find the following:

(a) $\mathbf{v} \cdot \mathbf{w}$.

(b) A unit vector in the same direction as $\mathbf{w}$.

(c) The cosine of the angle θ between $\mathbf{v}$ and $\mathbf{w}$.

(d) A nonzero vector orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

2. (15 points) Find an equation for the sphere centered at $(1, 1, 2)$ that passes through the point $(1, 0, -1)$.

3. (15 points) Find an equation for the plane that contains the point $(2, 3, 4)$ and also contains the line parametrized by $x = 2t$, $y = 2t + 1$, $z = 3t$.

4. (12 points) Mark the following statements as either true or false. (There is no partial credit or penalty for guessing.) All statements take place in 3-dimensional space.

(a) **True** **False** Two distinct planes either intersect or are parallel.

(b) **True** **False** Two distinct lines either intersect or are parallel.

(c) **True** **False** Two distinct planes each parallel to a given line are parallel.

(d) **True** **False** Two distinct planes each perpendicular to a given plane are parallel.

5. (12 points) At time t, a particle moving through space has acceleration

$$\mathbf{a}(t) = \langle -3\cos t, -3\sin t, 2 \rangle.$$

Also, its initial velocity and position are, respectively,

$$\mathbf{v}(0) = 3\mathbf{j}$$
$$\mathbf{r}(0) = 3\mathbf{i}.$$

Find the particle's velocity $\mathbf{v}(t)$ and position $\mathbf{r}(t)$ at time t.

6. (15 points) Let C be the circle of radius a parametrized by $\mathbf{r}(t) = (a\cos t)\mathbf{i} + (a\sin t)\mathbf{j}$.

(a) Find the unit tangent vector $\mathbf{T}(t)$ at time t.

(b) Compute the curvature $\kappa = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ of this circle.

(c) Assume that the equator of Earth is a perfect circle with diameter 7918 miles. Find the curvature of the equator.

7. (15 points) For each of the given limits, either evaluate it or show that it does not exist:

(a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{4}{\sqrt{4e^{x^2+y^2} + (\cos(xy))^2 + 1}}$

(b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{6yx^3}{2x^4 + y^4}$

Blank page for scratch work