# Math 164: Multidimensional Calculus

Final Exam

December 15, 2015

**NAME (please print legibly):** ______________________________

**Your University ID Number:** ______________________________

**Indicate your instructor with a check in the appropriate box:**

| | | |
|---|---|---|
| Bobkova | TR 12:30-1:45pm | |
| Chen | MW 3:25-4:40pm | |
| Dummit | TR 3:25-4:40pm | |
| Salur | MWF 09:00-09:50am | |

- You are responsible for checking that this exam has all 19 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers. Please sign the pledge below.

## Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

**Signature:** ______________________________

| Part A | | |
|---|---|---|
| QUESTION | VALUE | SCORE |
| 1 | 14 | |
| 2 | 12 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 12 | |
| 6 | 16 | |
| 7 | 16 | |
| 8 | 10 | |
| TOTAL | 100 | |

| Part B | | |
|---|---|---|
| QUESTION | VALUE | SCORE |
| 9 | 14 | |
| 10 | 14 | |
| 11 | 12 | |
| 12 | 12 | |
| 13 | 12 | |
| 14 | 12 | |
| 15 | 12 | |
| 16 | 12 | |
| TOTAL | 100 | |

**Part A**

**1. (14 points)** Consider the two planes $4x + y - z = 4$ and $x + 4y - z = 1$.

(a) Find the (acute) angle between the planes.

(b) Find a parametrization for the line of intersection of these two planes.

**2. (12 points)** Find the distance between the two skew lines whose parametrizations are

$$
\begin{aligned}
l_1 : \langle x, y, z \rangle &= \langle 1, 0, 0 \rangle + t \langle -2, 1, 0 \rangle \\
l_2 : \langle x, y, z \rangle &= \langle 0, 1, 1 \rangle + s \langle 2, -1, 1 \rangle .
\end{aligned}
$$

**3. (10 points)** A particle travels a total distance of $26\pi$ along the parametric curve

$$\mathbf{r}(t) = (5\sin t)\mathbf{i} + (5\cos t)\mathbf{j} + 12t\mathbf{k}$$

from the starting point $(0, -5, 12\pi)$. Find its new location.

**4. (10 points)** Evaluate the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{x^4+y^6}$$

or show that it does not exist.

**5. (12 points)** Find the linearization of the function

$$f(x,y) = \sqrt{12 - x^2 - 7y^2}$$

at $(x,y) = (2,1)$, and then use it to find the approximate value of $f(2.05, 0.98)$.

**6. (16 points)** Find the critical points of the function

$$f(x, y) = x^2y - 8y^2 - x^2$$

and classify each of them as a local minimum, local maximum, or saddle point.

**7. (16 points)** Calculate the integral $\iint_R y\,dA$ where $R$ is the (finite) region lying between the curves $x = y^2$ and $y = x - 2$.

**8. (10 points)** Circle the correct response for the following questions (no work is required, and there is no partial credit):

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(a) The result obtained by reversing the order of integration in the iterated double integral

$$\int_0^1 \int_x^2 x\, dy\, dx$$

is

(i) $\int_0^1 \int_x^2 x\, dx\, dy$

(ii) $\int_0^2 \int_0^y x\, dx\, dy$

(iii) $\int_x^2 \int_0^1 x\, dx\, dy$

(iv) $\int_0^1 \int_y^2 x\, dx\, dy + \int_1^2 \int_0^y x\, dx\, dy$

(v) $\int_0^1 \int_0^y x\, dx\, dy + \int_1^2 \int_0^1 x\, dx\, dy$

(vi) $\int_0^1 \int_0^2 x\, dx\, dy + \int_1^2 \int_y^2 x\, dx\, dy$

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(b) In polar coordinates, the integral

$$\int_{-2}^0 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x\, dy\, dx$$

is

(i) $\int_{\pi}^{2\pi} \int_0^2 r^2 \cos\theta\, dr\, d\theta$

(ii) $\int_{\pi/2}^{3\pi/2} \int_0^2 r^2 \cos\theta\, dr\, d\theta$

(iii) $\int_0^{2\pi} \int_0^2 r^2 \cos\theta\, dr\, d\theta$

(iv) $\int_{-\pi/2}^{\pi/2} \int_0^2 r \cos\theta\, dr\, d\theta$

(v) $\int_{\pi/2}^{3\pi/2} \int_0^2 r \cos\theta\, dr\, d\theta$

(vi) $\int_0^{2\pi} \int_0^2 r \cos\theta\, dr\, d\theta$

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**Part B**

**9. (14 points)** Evaluate the triple integral $\iiint_E x \, dV$, where $E$ is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$, whose vertices are $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

**10. (14 points)** Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx.$$

**11. (12 points)** Evaluate the line integral $\int_C (x - y + z + 2)\, ds$, where $C$ is the straight line segment from $(0, 1, 1)$ to $(1, 0, 1)$.

**12. (12 points)** Let $f$ be a scalar function and $\mathbf{F}$ be a vector field. For each expression, identify whether it is a scalar function, a vector field, or nonsense by circling the appropriate response (no work is required, and there is no partial credit).
Note that $\text{grad}(f) = \nabla f$ denotes the gradient of $f$.

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- $\text{curl}(f)$ **Scalar function** **Vector field** **Nonsense**
- $\text{grad}(f)$ **Scalar function** **Vector field** **Nonsense**
- $\text{div}(\mathbf{F})$ **Scalar function** **Vector field** **Nonsense**
- $\text{curl}(\text{grad}(f))$ **Scalar function** **Vector field** **Nonsense**
- $\text{grad}(\mathbf{F})$ **Scalar function** **Vector field** **Nonsense**
- $\text{grad}(\text{div}(\mathbf{F}))$ **Scalar function** **Vector field** **Nonsense**
- $\text{div}(\text{grad}(f))$ **Scalar function** **Vector field** **Nonsense**
- $\text{grad}(\text{div}(f))$ **Scalar function** **Vector field** **Nonsense**
- $\text{curl}(\text{curl}(\mathbf{F}))$ **Scalar function** **Vector field** **Nonsense**
- $\text{div}(\text{div}(\mathbf{F}))$ **Scalar function** **Vector field** **Nonsense**
- $(\text{grad}(f)) \times (\text{div}(\mathbf{F}))$ **Scalar function** **Vector field** **Nonsense**
- $\text{div}(\text{curl}(\text{grad}(f)))$ **Scalar function** **Vector field** **Nonsense**

**13. (12 points)** Find all possible values of the constants $a$ and $b$ such that the vector field

$$\mathbf{F}(x, y, z) = (2bxz^3 + ayz + 2xy)\mathbf{i} + (2by + ax^2 + axz)\mathbf{j} + (axy + bz + bx^2z^2)\mathbf{k}$$

is conservative (i.e., that the work done by the field on a particle moving through space does not depend on the particle's path).

**14. (12 points)** Compute the work integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F}(x, y) = \left(x^{2015} + x^2 y\right) \mathbf{i} + \left(xy^2 + 2e^y\right) \mathbf{j}$$

and the closed path $C$, oriented counterclockwise, consists of the following three pieces:

$$\begin{aligned}
C_1 &: \text{ the line segment from } (0,0) \text{ to } \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \\
C_2 &: \text{ the curve } y = \sqrt{1-x^2} \text{ with } -\frac{\sqrt{2}}{2} \le x \le \frac{1}{2}, \\
C_3 &: \text{ the line segment from } \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ to } (0,0).
\end{aligned}$$

**15. (12 points)** Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 4$.

**16. (12 points)** Compute the flux of the vector field

$$\mathbf{F}(x, y, z) = \left(-3x^2y\right)\mathbf{i} + (z - y)\,\mathbf{j} + (2x)\,\mathbf{k}$$

through the surface $S$ given by the part of the plane $z = 1 + 2x + y$ lying above the rectangle $0 \le x \le 2$, $0 \le y \le 3$, with upward orientation.

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