

Some Formulas

- $\text{comp}_{\mathbf{u}}(\mathbf{v}) = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|}, \quad \text{proj}_{\mathbf{u}}(\mathbf{v}) = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|^2} \mathbf{u}, \quad D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$
- $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n} dS = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) dA, \quad \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_C P dx + Q dy$
- $\iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}, \quad \iiint_E \text{div} \mathbf{F} dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$

For spherical coordinates (ρ, θ, ϕ) with $0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi$:

- $dV = \rho^2 \sin \phi d\rho d\phi d\theta,$
- For $\mathbf{r}(\phi, \theta) = a \sin \phi \cos \theta \mathbf{i} + a \sin \phi \sin \theta \mathbf{j} + a \cos \phi \mathbf{k}$:

$$\mathbf{r}_\phi \times \mathbf{r}_\theta = a^2 \sin^2 \phi \cos \theta \mathbf{i} + a^2 \sin^2 \phi \sin \theta \mathbf{j} + a^2 \sin \phi \cos \phi \mathbf{k}$$

Trig Identities

- $\sin^2 \theta + \cos^2 \theta = 1, \quad \tan^2 \theta + 1 = \sec^2 \theta, \quad \cot^2 \theta + 1 = \csc^2 \theta$
- $\sin(2\theta) = 2 \sin \theta \cos \theta$
- $\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta)), \quad \cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$
- $\sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b), \quad \sin(a - b) = \sin(a) \cos(b) - \cos(a) \sin(b)$
- $\cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b), \quad \cos(a - b) = \cos(a) \cos(b) + \sin(a) \sin(b)$
- $\sin(a) \cos(b) = \frac{1}{2}[\sin(a - b) + \sin(a + b)]$
- $\sin(a) \sin(b) = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$
- $\cos(a) \cos(b) = \frac{1}{2}[\cos(a - b) + \cos(a + b)]$