Answers to Math 164 Midterm 1, Fall 2014.

1. a) $\mathbf{a}\cdot\mathbf{b} = (0)(1) + (5)(1) + (-3)(1) = 2$.
   b) Scalar component is $\dfrac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||} = \dfrac{2}{\sqrt{0^2+5^2+(-3)^2}} = \dfrac{2}{\sqrt{34}}$.
   c) Vector projection is $\dfrac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\mathbf{a} = \dfrac{2}{34}(5\mathbf{j} - 3\mathbf{k})$.
2. Direction vectors are $\langle 2,-1,3\rangle$ and $\langle 2,-1,3\rangle$ so they are parallel. For distance, choose points A and B on each line, and then compute the length of the component of $\mathbf{w} = AB$ that is orthogonal to the direction vector $\mathbf{v}$. Easy points are $(1,-1,0)$ and $(5,1,8)$ yielding $\mathbf{w} = \langle 4,2,8\rangle$. Then the orthogonal component is $\mathbf{w} - \dfrac{\mathbf{w}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v} = \langle 4,2,8\rangle - \dfrac{30}{14}\langle 2,-1,3\rangle = \dfrac{1}{7}\langle -2,\ 29,\ 11\rangle$, and the length is $\dfrac{1}{7}\sqrt{(-2)^2+29^2+11^2} = \dfrac{\sqrt{966}}{7} = \sqrt{\dfrac{138}{7}}$.
3. a) Angle α is π minus angle between normal vectors $\mathbf{v} = \langle 3,0,6\rangle$ and $\mathbf{w} = \langle 2,2,-1\rangle$. Angle between normal vectors has $\cos(\theta) = \dfrac{\mathbf{v}\cdot\mathbf{w}}{||\mathbf{v}||\,||\mathbf{w}||} = \dfrac{0}{3\sqrt{45}} = 0$, so $\theta = \dfrac{\pi}{2}$. Thus, angle is $\pi - \dfrac{\pi}{2} = \dfrac{\pi}{2}$.
   b) Direction vector of line is $\mathbf{v}\times\mathbf{w} = \langle -12,15,6\rangle$. A point on both planes with $z=0$ is $(x,y,z) = (\frac{1}{3},\frac{7}{6},0)$ so line is $(\frac{1}{3} - 12t, \frac{7}{6} + 15t, 6t)$.
4. Velocity at time 0 is in direction of $\langle 2,1,1\rangle$ of magnitude 2 so $\mathbf{v}(0) = \dfrac{2}{\sqrt{6}}\langle 2,1,1\rangle$. Thus $\mathbf{v}(t) = \dfrac{2}{\sqrt{6}}\langle 2,1,1\rangle + \langle 2,1,1\rangle\, t$ by integrating $\mathbf{a}(t)$. Integrating again gives $\mathbf{r}(t) = \langle 1,-1,2\rangle + \dfrac{2}{\sqrt{6}}\langle 2,1,1\rangle\, t + \dfrac{1}{2}\langle 2,1,1\rangle\, t^2$ since $\mathbf{r}(0) = \langle 1,-1,2\rangle$.
5. $\mathbf{r}'(t) = \langle t\cos t,\ t\sin t\rangle$ so $||\mathbf{r}'(t)|| = |t|$, $\mathbf{T}(t) = \langle \cos t, \sin t\rangle$, $\mathbf{T}'(t) = \langle -\sin t, \cos t\rangle$, $||\mathbf{T}'(t)|| = 1$. So $\kappa(t) = \dfrac{1}{|t|}$.
6. a) Level surface is $\ln(x^2+y^2+z^2) = \ln(4)$, or $x^2+y^2+z^2 = 4$. This is the sphere of radius 2 centered at $(0,0,0)$.
   b) Level surface is $\dfrac{x-y+z}{2x+y-z} = \dfrac{-1}{4}$, or $6x - 3y + 3z = 0$. This is a plane with normal vector $\langle 6,3,-3\rangle$ passing through $(0,0,0)$.