Answers to Math 164 Final, Fall 2012.

1. a) $2(x-1)+2(y-1)-2\sqrt{3}(z-\sqrt{3})=0$, or $2x+2y-2\sqrt{3}z=-2$.
   b) Intersection is $(2,2,0)$ and distance is $\sqrt{5}$.

2. The limit does not exist: the limit along $x=0$ is ∞ while the limit along $y=x$ is $\frac{1}{2}$.

3. Minimum is -3 and maximum is $+3$, occurring at $(x,y)=(\pm 1,\pm 3)$.

4. $(0,0)$ is a saddle point, $(1,0)$ is a local minimum.

5. 6π.

6. a) $\frac{1}{2\pi}\left(2\pi+\frac{32}{3}\right)=1+\frac{16}{3\pi}$.
   b) -16.
   c) Yes, it is conservative: $\mathbf{F}=\nabla U$ where $U=x^3+\frac{1}{2}y^2$.
   d) $U(3,4)-U(-1,0)=36$.

7. By Green's Theorem, it is $\iint_D [y\,e^x - y\,e^x + 2]\;dy\,dx = \iint_D 2\,dy\,dx = 2\,\text{Area}(D) = 8$.

8. $\frac{1}{3}\left(2^{3/2}-1\right)\pi$.

9. a) $5z$.
   b) 32.
   c) By the Divergence Theorem it is $\int_0^2\int_0^2\int_0^2 5z\,dz\,dy\,dx = 40$.
   d) $40-32=8$.

10. a) $\langle xe^{xy},\, -ye^{xy},\, 4\rangle$.
    b) $\langle x,y,z\rangle = \langle \cos t,\, \sin t,\, 0\rangle$ for $0\le t\le 2\pi$.
    c) By Stokes's Theorem, it is $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds = \int_0^{2\pi}\left[-\sin t\cdot(-\sin t)+(3\cos t)(\cos t)+0\right]\,dt = 4\pi$.