MATH 164

Midterm 2

November 17, 2011

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Circle your Instructor's Name along with the Lecture Time:

Carl Mueller (9:00am)   Mihai Bailesteanu (10:00am)

- **No calculators are allowed on this exam.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please put your simplified final answers in the spaces provided.**

QUESTION	VALUE	SCORE
1	12	
2	13	
3	12	
4	13	
5	13	
6	12	
7	12	
8	13	
TOTAL	100	

1. (12 points) Consider the function $f(x, y) = x^2 - y^3 + 3y + 1$ for $(x, y) \in \mathbf{R}^2$.

(a) Find the critical points of f.

(b) Find the local maxima, minima, and saddle points of f.

(c) Are any of these points global maxima or minima?

2. (13 points) Find the extreme values of $f(x,y) = x^2 + xy + y^2 + x - y + 1$ subject to the constraint $g(x,y) = x^2 + y^2 = 1$. Go through the following steps:

(a) Use the Lagrange multiplier method to get two equations involving x, y and λ (the Lagrange multiplier).

(b) Starting from these two equations, show that $\lambda = \frac{3}{2}$ or $y = \frac{1}{1-2\lambda}$ and provide an argument to show $\lambda \neq \frac{1}{2}$.

(c) Find the four extreme points and evaluate f at these points to see which ones are maximum points and which ones are minimum points.

3. (12 points) Consider the following integral:

$$I = \int_1^3 \int_1^2 xye^{xy^2} dxdy$$

Is it easier to solve this integral as written, or by changing the order of integration? Choose the easiest way and evaluate the integral.

4. (13 points) Using polar coordinates, calculate the integral

$$\iint_D (x^2 + y^2)\ dx\ dy$$

where the domain D is inside the half-circle $x^2 + y^2 = 1$, $x \geq 0$, between the lines $y = \sqrt{3}x$, $x = \sqrt{3}y$. Go through the following steps:

(a) Determine the bounds for r and θ (it might be helpful to draw a sketch of the domain).

(b) Solve the new double integral.

5. (13 points)

Find the center of mass of the region between the curve $y = x^2$, the x-axis, and the line $y = 1$.

(a) Assuming the density of region is 1, what is the mass of the region?

(b) Find the center of mass $\bar{x}$ in the x direction.

(c) Find the center of mass $\bar{y}$ in the y direction.

6. (12 points)

Calculate the following triple integral:

$$I = \iiint_E \frac{dx\ dy\ dz}{(1+x+y+z)^3}$$

where E is the domain bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

7. (12 points)

In this question we will set up an integral in cylindrical coordinates. **DO NOT EVALUATE THE INTEGRAL**.

The problem is to determine the volume of the region inside the sphere $x^2 + y^2 + z^2 = 2$ and above the plane $z = 1$.

(a) First find the value of r at the points of intersection of the sphere and the plane.

(b) Find the upper and lower limits of integration for z, as a function of r.

(c) Set up the integral, but **DO NOT EVALUATE IT**.

8. (13 points) Using spherical coordinates, calculate the integral

$$I = \iiint_E \sqrt{x^2 + y^2 + z^2}\ dx\ dy\ dy$$

where $E = \{(x, y, z) \in \mathbb{R} : z \geq 0, x^2 + y^2 + z^2 \leq z\}$. Follow these steps:

(a) Knowing that $0 < \rho < g(\phi)$ and $0 \leq \phi \leq h(\pi)$, determine the expressions $g(\phi)$ and $h(\pi)$. Are there any restrictions for θ?

(b) Calculate I and show that $I < \frac{1}{3}$.