MATH 164

Final

December 17, 2011

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Circle your Instructor's Name along with the Lecture Time:

Mueller (9:00am)   Bailesteanu (10:00am)

- **Part A of the final can replace a bad midterm. However, Part A will still count towards your score on the final. If you skip part A, you will get a very low score on the final and probably also in the course.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**

Part A		
QUESTION	VALUE	SCORE
1	9	
2	8	
3	9	
4	8	
5	8	
6	8	
TOTAL	50	

Part B		
QUESTION	VALUE	SCORE
7	8	
8	8	
9	9	
10	8	
11	9	
12	8	
TOTAL	50	

Part A

1. (9 points) Find an equation for the plane passing through the points

$$(1, 2, 3) \qquad (1, 0, -2) \qquad (0, -2, 1)$$

2. (8 points) Calculate the following double integral:

$$\iint_D \sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9}}\, dA$$

where $D = \left\{ (x, y) \in \mathbb{R}^2 \middle| \frac{x^2}{4} + \frac{y^2}{9} \leq 1 \right\}$ (the interior of an ellipse).

Follow these steps:

(a) Use the substitution $x = 2r\cos\theta$ and $y = 3r\sin\theta$. Write the domain E of r and θ and calculate the Jacobian $J(r, \theta) = \frac{\partial(x,y)}{\partial(r,\theta)}$ of this change of variables.

(b) Now calculate

$$\iint_D f(x, y)\, dA = \iint_D \sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9}}\, dA = \iint_E f(x(r, \theta), y(r, \theta))\, |J(r, \theta)|\, dr\, d\theta.$$

3. (9 points) Suppose that

$$\frac{\partial f}{\partial x}(1,2) = 7$$
$$\frac{\partial f}{\partial y}(1,2) = -3$$

and

$$x(t) = t^2$$
$$y(t) = 2t^3$$

Find

$$\frac{d}{dt} f(x(t), y(t))\Big|_{t=1}$$

4. (8 points)

A solid is bounded by the four planes given below. Its density is given by the function $f(x, y, z) = \sin(x + y + z)$. Calculate the mass of the solid.

(1) $x = 0$

(2) $y = 0$

(3) $z = 0$

(4) $x + y + z = \frac{\pi}{2}$

5. (8 points) Consider the function

$$f(x, y) = x^2 - 3\frac{x}{y} + y^3$$

Find a vector v which is tangent to the level curve of the surface $z = f(x, y)$ at the point $(x, y) = (2, -1)$.

6. (8 points)

Consider a particle whose position is given by the curve $\mathbf{r}(t) = e^t \cos t \cdot \mathbf{i} + e^t \sin t \cdot \mathbf{j} + te^t \cdot \mathbf{k}$, for $t \in [0, 1]$.

(a) Calculate its velocity, acceleration and speed.

(b) Write the integral which gives the total distance traveled by the particle. **Do not evaluate it**.

Part B

7. (8 points) Suppose that

$$u = xe^{xy}$$
$$v = x^2 + y^2$$

Let D, E be regions in $\mathbf{R}^2$ such that if we change variables from (x, y) to (u, v) and $x, y \in D$, then $(u, v) \in E$. Furthermore, the map between the two regions is one to one. Then we can write

$$\iint\limits_E f(u, v)dudv = \iint\limits_D f(xe^{xy}, x^2 + y^2)h(x, y)dxdy.$$

Find $h(x, y)$.

8. (8 points)

Let $\mathbf{F}$ be a vector field defined as follows:

$$\mathbf{F} = x^3\ \mathbf{i} + y^3\ \mathbf{j} + z^3\ \mathbf{k}$$

and S be the sphere given by the equation $x^2 + y^2 + z^2 = 4$.

(a) Calculate div $\mathbf{F}$.

(b) Write down a suitable parametrization of the surface S. In this parametrization, write the expression of $\iint_S \mathbf{F} \cdot\ d\mathbf{S}$.
Do not evaluate it yet.

(c) State the divergence theorem in this setting, decide if it is useful for evaluating

$$\iint_S \mathbf{F} \cdot\ d\mathbf{S}.$$

Finish this exercise by calculating the above integral.

9. (9 points) Suppose that

$$\mathbf{F}(x, y) = (\sin x \cos y, x^2 y)$$

Let D be the interior of the unit square with vertices at $(0,0), (0,1), (1,0), (1,1)$.

(a) Write Green's theorem for this vector field and this region. Find specific integrals for both the line integral and the double integral. **Do not evaluate**.

(b) Evaluate the line integral from part (a).

10. (8 points)

Calculate the area of the surface S given by the equation $z = \frac{1}{2}xy$, which lies inside the cylinder $x^2 + y^2 = 4$. Follow the steps:

(a) Write a suitable parametrization of the surface, calculate the vector $\mathbf{r}_x \times \mathbf{r}_y$ and its length.

(b) Determine the domain D of the parameters, i.e. determine what is the projection of the surface onto the (xy)-plane and show that $\text{Area}(S) = \iint_D \frac{1}{2}\sqrt{4 + x^2 + y^2}\, dx\, dy$.

(c) Calculate the integral from above. **Hint:** you may want to change coordinates.

11. (9 points) Suppose that S is the surface defined by $z = 1 - x^2 - y^2$ for $z \geq 0$, and C is the boundary of this surface. Let

$$\mathbf{F}(x, y, z) = x\mathbf{i} + z\mathbf{j} - 3y\mathbf{k}.$$

(a) What does Stokes theorem say about this situation? First write down what Stokes theorem says in general, using the symbols S, C, $\mathbf{F}$.

(b) Evaluate the line integral which occurs in Stokes theorem.

(c) Find curl($\mathbf{F}$).

(d) Let r, θ be polar coordinates in the x-y plane. Let $R(r, \theta)$ be a parameterization of the surface. Find R_r, R_θ, and a normal vector to the surface (not necessarily a unit vector).

12. (8 points)

Show that the vector field $\mathbf{F} = (y+z)\,\mathbf{i} + (x+z)\,\mathbf{j} + (x+y)\,\mathbf{k}$ is conservative and find the function f such that $\mathbf{F} = \nabla f$. Calculate $\int_C \mathbf{F} \cdot \, d\mathbf{s}$ where C is a simple curve connecting the points $P_1 = (0,1,2)$ and $P_2 = (1,2,3)$.