MATH 164

PRACTICE FINAL

Tuesday December 16th

4:00pm – 7:00pm
(3 hours)

- No calculators are allowed on this exam.
- A single 'crib sheet' will be allowed.
- Please show all of your work. An answer without the reasons given will receive 0 points.
- Please indicate your final answer CLEARLY!

1. Let $P = (1, 0, 0)$, $Q = (0, 1, 0)$, $R = (0, 0, -1)$

a. Find the equation of the plane passing through P, Q and R

b. Find the orthogonal projection of the vector $(1, 2, 3)$ onto the plane passing through P, Q and R.

c. Find the distance between $(1, 2, 3)$ and the plane passing through P, Q and R.

2. Find the volume of the parallelepiped determined by the vectors $\vec{i} + \vec{j}$, $\vec{j} + \vec{k}$, $\vec{i} + \vec{k}$.

3. Reparametrize the curve $\vec{r}(t)$ with respect to arc-length from the point where $t = 0$. $\vec{r}(t) = \langle e^t \cos t, e^t \sin t \rangle$

4. Let $f(x, y) = \frac{x}{y}$.

a. Find the linear approximation of $f(x, y)$ at $(6, 3)$.

b. Approximate $\frac{5.8}{2.9}$ using the above linear approximation.

5. Find and classify the critical points of the function $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$

6. Find the maximum and minimum values of the function $f(x, y, z) = x^4 + y^4 + z^4$ subject to the constraints $x^2 + y^2 + z^2 = 1$.

7. Compute the volume of the region under the paraboloid $z = x^2 + y^2$ and above the disc $x^2 + y^2 \leq 9$.

8. Find center of mass of the region D in the plane, where D is bounded by the parabola $y = 9 - x^2$ and the x-axis, with $\rho(x, y) = y$.

9. Evaluate $\int\int\int_E z dV$ where E lies between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$ in the first octant.

10. Consider the vector field $\vec{F}(x, y, z) = \langle yze^{xz}, e^{xz}, xye^{xz} \rangle$

a. Is $\vec{F}$ conservative? If so, find a potential function for $\vec{F}$.

b. What is the integral $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve that satisfies the equations $x^2 + \frac{y^2}{2} = 1$ and $x + y + z = 0$. (orient the curve so that when one looks 'down' on the curve from the z-axis, it is oriented in a counter-clockwise fashion.)

11. Evaluate the integral $\int_C e^y dx + 2xe^y dy$ where C is the square with sides $x = 1$, $x = 3$, $y = 1$, $y = 4$ (oriented CW).

12. Let $\vec{F}(x, y, z) = \langle y, z, x \rangle$. Compute the flux integral $\int\int_S \vec{F} \cdot d\vec{S}$ where S is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$. Orient S so that the components of the normal vector are all positive.

13. Let the surface S be the graph of the function $z = x^2 - y^2$ with domain $D = \{(x, y) : -1 \leq x \leq 1, -1 \leq y \leq 1\}$. Give S an 'upward' orientation. Let $\vec{F}$ be the vector-field $\vec{F}(x, y, z) = \langle z, x, y \rangle$ Compute $\int\int_S \vec{F} \cdot d\vec{S}$ (the flux of $\vec{F}$ across the surface S).

14. Compute the flux integral

$$\int\int_S \vec{F} \cdot d\vec{S}$$

where $\vec{F} = \langle -xz, z, z^2 \rangle$ and S is the outwardly oriented boundary of the solid from problem 9.

15. Verify Stoke's Theorem for the vector field $\vec{F}(x, y, z) = \langle 3y, 4z, -6x \rangle$ and the surface S which is the part of the paraboloid $z = 9 - x^2 - y^2$ over the xy-plane, oriented upward.