

Math 164: Multidimensional Calculus

Final Exam

December 15, 2015

NAME (please print legibly): Solutions

Your University ID Number: _____

Indicate your instructor with a check in the appropriate box:

Bobkova	TR 12:30-1:45pm	<input type="checkbox"/>
Chen	MW 3:25-4:40pm	<input type="checkbox"/>
Dummit	TR 3:25-4:40pm	<input type="checkbox"/>
Salur	MWF 09:00-09:50am	<input type="checkbox"/>

- You are responsible for checking that this exam has all 19 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers. Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: _____

Part A		
QUESTION	VALUE	SCORE
1	14	
2	12	
3	10	
4	10	
5	12	
6	16	
7	16	
8	10	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
9	14	
10	14	
11	12	
12	12	
13	12	
14	12	
15	12	
16	12	
TOTAL	100	

Part A

1. (14 points) Consider the two planes $4x + y - z = 4$ and $x + 4y - z = 1$.

(a) Find the (acute) angle between the planes.

$$\vec{n}_1 = \langle 4, 1, -1 \rangle$$

$$\vec{n}_2 = \langle 1, 4, -1 \rangle$$

$$\vec{n}_1 \cdot \vec{n}_2 = |\vec{n}_1| \cdot |\vec{n}_2| \cdot \cos \theta$$

$$4 + 4 + 1 = \sqrt{4^2 + 1^2 + (-1)^2} \cdot \sqrt{1^2 + 4^2 + (-1)^2} \cdot \cos \theta$$

$$\frac{9}{18} = \cos \theta$$

$$\frac{1}{2} = \cos \theta$$

$$\theta = 60^\circ$$

(b) Find a parametrization for the line of intersection of these two planes.

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -1 \\ 1 & 4 & -1 \end{vmatrix} = 3\hat{i} + 3\hat{j} + 15\hat{k}$$

Take a point from the intersection: $x = 0$

$$\begin{cases} y - z = 4 \\ 4y - z = 1 \end{cases}$$

$$-y + z = -4$$

$$4y - z = 1$$

$$3y = -3$$

$$y = -1$$

$$z = -5$$

$$\left. \begin{aligned} x(t) &= 0 + 3t \\ y(t) &= -1 + 3t \\ z(t) &= -5 + 15t \end{aligned} \right\} \text{A parametr. for the line of intersection.}$$

