Math 164: Multidimensional Calculus

Final Exam

December 17, 2016

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Indicate your instructor with a check in the appropriate box:

Kleene	TR 12:30-1:45pm	
Salur	MW 3:25-4:40pm	
Gafni	TR 3:25-4:40pm	
Lee	MWF 09:00-09:50am	

- You are responsible for checking that this exam has all 15 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers. Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

Part A		
QUESTION	VALUE	SCORE
1	18	
2	16	
3	16	
4	16	
5	18	
6	16	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
7	16	
8	16	
9	16	
10	18	
11	16	
12	18	
TOTAL	100	

Part A

1. (18 points)

Consider the vectors

$$\mathbf{a} = \langle 1, -1, 3 \rangle, \quad \mathbf{b} = \langle -2, 1, 1 \rangle, \quad \mathbf{c} = \langle 1, 0, 5 \rangle.$$

Compute the following.

(a) The angle between **a** and **b**.

(b) The projection of **b** onto **c**.

(c) The area of the parallelogram spanned by **a** and **c**.

2. (16 points) For each of the following statements, circle TRUE or FALSE. No work is required, and there is no partial credit.

(a) The curve $\mathbf{r}(t) = \langle t^3, -t^3, 2t^3 \rangle$ is a line. TRUE FALSE

(b) If $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are differentiable vector functions,
then $\frac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}'(t)$. TRUE FALSE

(c) If $|\mathbf{r}(t)| = 1$ for all t then $|\mathbf{r}'(t)| = 0$. TRUE FALSE

(d) The curve $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$, $0 \le t \le 1$, has arclength 4π. TRUE FALSE

3. (16 points) Find the limit, if it exists, or show that the limit does not exist.

(a)

$$\lim_{(x,y)\to(1,1)} \frac{e^x \ln y}{x^2 + 2y^2}$$

(b)

$$\lim_{(x,y)\to(0,0)} \frac{x \sin y}{x^2 + 2y^2}$$

4. (16 points) (a) Find the equation of the tangent plane to the surface $z = x^2 + 2y^2$ at the point (2,0,1).

(b) What is an approximate value of $f(2.1, -0.1)$ when $f(x, y) = x^2 + 2y^2$?

5. (18 points) Find the extreme values of the function $f(x, y) = xy$ over the curve $x^2 + y^4 = 3/4$.

6. (16 points) Evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_x^1 e^{x/y} dy dx$$

Part B

7. (16 points) Find the volume of the solid that lies inside the sphere $x^2 + y^2 + z^2 = 4$ and outside the cylinder $x^2 + y^2 = 1$.

8. (16 points) Evaluate the line integral $\int_C xyz\, ds$, where C is the line segment from $(1, 2, 3)$ to $(2, 4, 5)$.

9. (16 points) Let T be the triangle with vertices $(1,0)$, $(1,1)$ and $(1,0)$, let $\mathbf{F}$ be the vector field given by

$$\mathbf{F}(x,y) = \langle xy^2\sin(x^2) + 4yx^2,\ -y\cos(x^2)\rangle.$$

Compute $\oint_{\partial T} \mathbf{F}\cdot d\mathbf{r}$.

10. (18 points) (a) Find a potential function for the vector field

$$\mathbf{F}(x, y, z) = \langle yz + 2xy,\ xz + x^2,\ xy + 4z \rangle$$

(b) Evaluate the line integral $\int_C \mathbf{F} \cdot \mathbf{dr}$, where C is the oriented curve parametrized by $r(t) = \langle t, t^2, t^4 - 1 \rangle$ for $0 \leq t \leq 1$.

11. (16 points) Evaluate the surface integral $\iint x^2yz\,dS$, where the surface S is the part of the plane $z = 1 + 2x + 3y$ that lies above the rectangle $[0, 3] \times [0, 2]$.

12. (18 points) Compute the flux of the vector field

$$\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}.$$

through the surface S given by the boundary of the solid region E enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$. Here S is given the positive (outward) orientation with respect to E.

No test material on this page.

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