Math 164: Multidimensional Calculus

Final Exam

December 15, 2015

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Indicate your instructor with a check in the appropriate box:

Bobkova	TR 12:30-1:45pm	
Chen	MW 3:25-4:40pm	
Dummit	TR 3:25-4:40pm	
Salur	MWF 09:00-09:50am	

- You are responsible for checking that this exam has all 19 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers. Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

Part A		
QUESTION	VALUE	SCORE
1	14	
2	12	
3	10	
4	10	
5	12	
6	16	
7	16	
8	10	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
9	14	
10	14	
11	12	
12	12	
13	12	
14	12	
15	12	
16	12	
TOTAL	100	

Part A

1. (14 points) Consider the two planes $4x + y - z = 4$ and $x + 4y - z = 1$.

(a) Find the (acute) angle between the planes.

(b) Find a parametrization for the line of intersection of these two planes.

2. (12 points) Find the distance between the two skew lines whose parametrizations are

$$\begin{aligned} l_1 : \langle x, y, z \rangle &= \langle 1, 0, 0 \rangle + t \langle -2, 1, 0 \rangle \\ l_2 : \langle x, y, z \rangle &= \langle 0, 1, 1 \rangle + s \langle 2, -1, 1 \rangle . \end{aligned}$$

3. (10 points) A particle travels a total distance of 26π along the parametric curve

$$\mathbf{r}(t) = (5\sin t)\mathbf{i} + (5\cos t)\mathbf{j} + 12t\mathbf{k}$$

from the starting point $(0, -5, 12\pi)$. Find its new location.

4. (10 points) Evaluate the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{x^4+y^6}$$

or show that it does not exist.

5. (12 points) Find the linearization of the function

$$f(x, y) = \sqrt{12 - x^2 - 7y^2}$$

at $(x, y) = (2, 1)$, and then use it to find the approximate value of $f(2.05, 0.98)$.

6. (16 points) Find the critical points of the function

$$f(x,y) = x^2y - 8y^2 - x^2$$

and classify each of them as a local minimum, local maximum, or saddle point.

7. (16 points) Calculate the integral $\iint_R y\,dA$ where R is the (finite) region lying between the curves $x = y^2$ and $y = x - 2$.

8. (10 points) Circle the correct response for the following questions (no work is required, and there is no partial credit):

(a) The result obtained by reversing the order of integration in the iterated double integral

$$\int_0^1 \int_x^2 x\,dy\,dx$$

is

(i) $\int_0^1 \int_x^2 x\,dx\,dy$

(ii) $\int_0^2 \int_0^y x\,dx\,dy$

(iii) $\int_x^2 \int_0^1 x\,dx\,dy$

(iv) $\int_0^1 \int_y^2 x\,dx\,dy + \int_1^2 \int_0^y x\,dx\,dy$

(v) $\int_0^1 \int_0^y x\,dx\,dy + \int_1^2 \int_0^1 x\,dx\,dy$

(vi) $\int_0^1 \int_0^2 x\,dx\,dy + \int_1^2 \int_y^2 x\,dx\,dy$

(b) In polar coordinates, the integral

$$\int_{-2}^0 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x\,dy\,dx$$

is

(i) $\int_{\pi}^{2\pi} \int_0^2 r^2 \cos\theta\,dr\,d\theta$

(ii) $\int_{\pi/2}^{3\pi/2} \int_0^2 r^2 \cos\theta\,dr\,d\theta$

(iii) $\int_0^{2\pi} \int_0^2 r^2 \cos\theta\,dr\,d\theta$

(iv) $\int_{-\pi/2}^{\pi/2} \int_0^2 r \cos\theta\,dr\,d\theta$

(v) $\int_{\pi/2}^{3\pi/2} \int_0^2 r \cos\theta\,dr\,d\theta$

(vi) $\int_0^{2\pi} \int_0^2 r \cos\theta\,dr\,d\theta$

Part B

9. (14 points) Evaluate the triple integral $\iiint_E x\, dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $x+y+z = 1$, whose vertices are $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

10. (14 points) Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx.$$

11. (12 points) Evaluate the line integral $\int_C (x - y + z + 2)\, ds$, where C is the straight line segment from $(0, 1, 1)$ to $(1, 0, 1)$.

12. (12 points) Let f be a scalar function and $\mathbf{F}$ be a vector field. For each expression, identify whether it is a scalar function, a vector field, or nonsense by circling the appropriate response (no work is required, and there is no partial credit).
Note that $\text{grad}(f) = \nabla f$ denotes the gradient of f.

• $\text{curl}(f)$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{grad}(f)$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{div}(\mathbf{F})$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{curl}(\text{grad}(f))$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{grad}(\mathbf{F})$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{grad}(\text{div}(\mathbf{F}))$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{div}(\text{grad}(f))$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{grad}(\text{div}(f))$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{curl}(\text{curl}(\mathbf{F}))$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{div}(\text{div}(\mathbf{F}))$	**Scalar function**	**Vector field**	**Nonsense**
• $(\text{grad}(f)) \times (\text{div}(\mathbf{F}))$	**Scalar function**	**Vector field**	**Nonsense**
• $\text{div}(\text{curl}(\text{grad}(f)))$	**Scalar function**	**Vector field**	**Nonsense**

13. (12 points) Find all possible values of the constants a and b such that the vector field

$$\mathbf{F}(x, y, z) = (2bxz^3 + ayz + 2xy)\mathbf{i} + (2by + ax^2 + axz)\mathbf{j} + (axy + bz + bx^2z^2)\mathbf{k}$$

is conservative (i.e., that the work done by the field on a particle moving through space does not depend on the particle's path).

14. (12 points) Compute the work integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F}(x, y) = \left(x^{2015} + x^2 y\right) \mathbf{i} + \left(xy^2 + 2e^y\right) \mathbf{j}$$

and the closed path C, oriented counterclockwise, consists of the following three pieces:

$$\begin{aligned} C_1 &: \text{ the line segment from } (0,0) \text{ to } \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \\ C_2 &: \text{ the curve } y = \sqrt{1-x^2} \text{ with } -\frac{\sqrt{2}}{2} \leq x \leq \frac{1}{2}, \\ C_3 &: \text{ the line segment from } \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ to } (0,0). \end{aligned}$$

15. (12 points) Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 4$.

16. (12 points) Compute the flux of the vector field

$$\mathbf{F}(x, y, z) = \left(-3x^2y\right)\mathbf{i} + (z - y)\,\mathbf{j} + (2x)\,\mathbf{k}$$

through the surface S given by the part of the plane $z = 1 + 2x + y$ lying above the rectangle $0 \le x \le 2$, $0 \le y \le 3$, with upward orientation.

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