Math 164: Multidimensional Calculus

Midterm 1

October 15, 2015

NAME (please print legibly): SOLUTIONS

Your University ID Number:

Indicate your instructor with a check in the appropriate box:

Bobkova	TR 12:30-1:45pm	
Chen	MW 3:25-4:40pm	
Dummit	TR 3:25-4:40pm	
Salur	MWF 09:00-09:50am	

- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 8 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers.
- Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature:

QUESTION	VALUE	SCORE
1	16	
2	15	
3	15	
4	12	
5	12	
6	15	
7	15	
TOTAL	100	

1. (16 points) Let $\mathbf{v} = \langle -4, 3, 0\rangle$ and $\mathbf{w} = \langle 2, -1, 2\rangle$. Find the following:

(a) $\mathbf{v} \cdot \mathbf{w}$.

$V \cdot W = -8 - 3 + 0 = -11$

(b) A unit vector in the same direction as $\mathbf{w}$.

$|W| = \sqrt{4+1+4} = 3$

$\vec{u} = \frac{\vec{W}}{|W|} = \langle \frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \rangle$

(c) The cosine of the angle θ between $\mathbf{v}$ and $\mathbf{w}$.

$|V| = \sqrt{16 + 9 + 0} = 5$

$\cos\theta = \frac{V \cdot W}{|V||W|} = \frac{-11}{15}$

(d) A nonzero vector orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

$V \times W = \begin{vmatrix} i & j & k \\ -4 & 3 & 0 \\ 2 & -1 & 2 \end{vmatrix} = 6i + 8j + (4-6)k = \langle 6, 8, -2 \rangle$

2. (15 points) Find an equation for the sphere centered at $(1, 1, 2)$ that passes through the point $(1, 0, -1)$.

$$(x-1)^2 + (y-1)^2 + (z-2)^2 = r^2$$

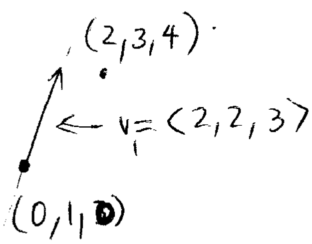
$$\cancel{(1-1)^2} + (0-1)^2 + (-1-2)^2 = r^2$$

$$1 + 9 = r^2$$

$$r = \sqrt{10}$$

$$(x-1)^2 + (y-1)^2 + (z-2)^2 = 10$$

3. (15 points) Find an equation for the plane that contains the point $(2, 3, 4)$ and also contains the line parametrized by $x = 2t$, $y = 2t + 1$, $z = 3t$.



$$V_1 = \langle 2, 2, 3 \rangle \qquad P = (2,3,4)$$

$$V_2 = \langle 2, 2, 4 \rangle$$

$$V_1 \times V_2 = \begin{vmatrix} i & j & k \\ 2 & 2 & 3 \\ 2 & 2 & 4 \end{vmatrix} = 2i - 2j + 0k$$

$$= \langle 2, -2, 0 \rangle = \vec{n}$$

$$\vec{n} \cdot \langle x-2, y-3, z-4 \rangle = 0$$

or $2(x-2) - 2(y-3) + 0(z-4) = 0$

$$2(x-2) - 2(y-3) = 0$$

$$2x - 2y - 4 + 6 = 0$$

$$\boxed{2x - 2y = -2}$$

4. (12 points) Mark the following statements as either true or false. (There is no partial credit or penalty for guessing.) All statements take place in 3-dimensional space.

(a) **True** **False** Two distinct planes either intersect or are parallel.

(b) **True** **False** Two distinct lines either intersect or are parallel.

(c) **True** **False** Two distinct planes each parallel to a given line are parallel.

(d) **True** **False** Two distinct planes each perpendicular to a given plane are parallel.

5. (12 points) At time t, a particle moving through space has acceleration

$$\mathbf{a}(t) = \langle -3\cos t, -3\sin t, 2\rangle .$$

Also, its initial velocity and position are, respectively,

$$\begin{aligned} \mathbf{v}(0) &= 3\mathbf{j} \\ \mathbf{r}(0) &= 3\mathbf{i}. \end{aligned}$$

Find the particle's velocity $\mathbf{v}(t)$ and position $\mathbf{r}(t)$ at time t.

$$\vec{v}(t) = \langle -3\sin(t), +3\cos(t), 2t\rangle \cancel{+\langle 0,0,0\rangle}$$

$$\vec{r}(t) = \langle +3\cos(t), 3\sin(t), t^2\rangle \cancel{+\langle 0,0,0\rangle}$$

6. (15 points) Let C be the circle of radius a parametrized by $\mathbf{r}(t) = (a\cos t)\mathbf{i} + (a\sin t)\mathbf{j}$.

(a) Find the unit tangent vector $\mathbf{T}(t)$ at time t. $r'(t) = (-a\sin t)\,i + (a\cos t)\,j \Rightarrow |r'(t)| = a$

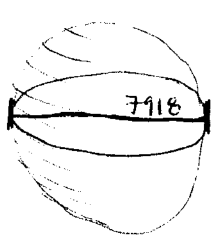
$T(t) = (-\sin t)\,i + (\cos t)\,j$

(b) Compute the curvature $\kappa = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ of this circle.

$T'(t) = (-\cos t)\,i + (-\sin t)\,j \Rightarrow |T'(t)| = 1$

$\Rightarrow \kappa(t) = \frac{1}{a}$

(c) Assume that the equator of Earth is a perfect circle with diameter 7918 miles. Find the curvature of the equator.



$K(\text{equator}) = \dfrac{2}{7918}$

$r = \frac{\text{diam}}{2}$

7. (15 points) For each of the given limits, either evaluate it or show that it does not exist:

(a) $\lim_{(x,y)\to(0,0)} \frac{4}{\sqrt{4e^{x^2+y^2}+(\cos(xy))^2+1}}$ $= \frac{4}{\sqrt{6}}$ all functions are cont's near $(0,0)$

(b) $\lim_{(x,y)\to(0,0)} \frac{6yx^3}{2x^4+y^4}$

Let C_1 be $y=0$. Then $\lim_{(x,y)\to(0,0)} \frac{6yx^3}{2x^4+y^4} = 0$ along C_1.

Let C_2 be $y=x$. Then $\lim_{(x,y)\to(0,0)} \frac{6yx^3}{2x^4+y^4} = \lim_{(x,y)\to(0,0)} \frac{6x^4}{2x^4+x^4} = 2$ along C_2

So $\boxed{\text{DNE}}$

Blank page for scratch work