Math 164: Multidimensional Calculus

Midterm 1

October 11, 2016

NAME (please print legibly): ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Your University ID Number: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Indicate your instructor with a check in the appropriate box:

Kleene	TR 12:30-1:45pm	
Salur	MW 3:25-4:40pm	
Gafni	TR 3:25-4:40pm	
Lee	MWF 09:00-09:50am	

- You have 75 minutes to work on this exam.

- You are responsible for checking that this exam has all 9 pages.

- No calculators, phones, electronic devices, books, notes are allowed during the exam.

- Show all work and justify all answers.

- Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

QUESTION	VALUE	SCORE
1	15	
2	20	
3	15	
4	10	
5	15	
6	15	
7	10	
TOTAL	100	

1. (15 points) Consider the vectors $a = <1, 0, 0>$, $b = <1, 0, -1>$ and $c = <1, 2, -1>$.

(a) Compute the scalar projection of b onto c.

(b) Find the angle between a and b.

(c) A vector $v = <5, y, z>$ satisfies $a \cdot v = b \cdot v = c \cdot v$. Find y and z.

2. (20 points) Consider the four points $P(0, 1, 5)$, $Q(1, 2, 8)$, $R(2, -1, 0)$, and $S(1, 2, 3)$

(a) Find an equation for the plane that passes through points P, Q, and R.

(b) Find the area of the triangle with vertices P, Q, and R.

(c) Find the volume of the parallelepiped determined by P, Q, R, and S.

(d) Find the distance from S to the plane determined by P, Q, and R.

3. (15 points) Let ℓ_1 be the line that passes through the points $(1, -2, 3)$ and $(2, 0, -1)$, and let ℓ_2 be the line that passes through the point $(3, 1, 2)$ and is perpendicular to the plane $x + 2y + 4z = 0$.

(a) Find symmetric and parametric equations for ℓ_1. Clearly label each set as symmetric or parametric.

(b) Find symmetric and parametric equations for ℓ_2. Clearly label each set as symmetric or parametric.

(c) Are these lines intersecting, parallel, or skew? Briefly explain your answer.

4. (10 points) Consider the curve $\mathbf{r}(t) = 2t\mathbf{i} + (3 - t)\mathbf{j} - 2t\mathbf{k}$.

(a) Find the arc length of the curve between $t = 0$ and $t = 1$.

(b) Reparametrize the curve in terms of arc length s measured from $t = 0$ in the direction of increasing t.

5. (15 points) Consider the curve $\mathbf{r}(t) = \cos 2t\mathbf{i} + \sin 2t\mathbf{j} + 2t\mathbf{k}$.

(a) Find the unit tangent vector $\mathbf{T}(t)$.

(b) Find the curvature $\kappa(t)$. Recall the curvature formula $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$.

6. (15 points) A gun has a muzzle speed of 80 meters per second.

(a) Assuming projectile motion, find the position function $r(t)$. Neglect air resistance and use $g = 9.8m/sec^2$ as the acceleration of gravity.

(b) What angle of elevation should be used to hit an object 200 meters away?

7. (10 points) Evaluate the limit or show that it does not exist.

(a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{4xy}{x^2 + 2y^2}$

(b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^3 + 5y^3}{x^2 + y^2}$

Hint: Use polar coordinates: $x = r\cos\theta$, $y = r\sin\theta$.

Blank page for scratch work