

Math 162 - Spring 2025

Workshop 7

March 17 - March 21

Area of a surface of revolution - Parametric curves

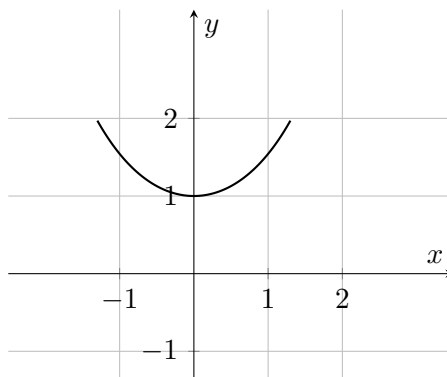
Problem 1.

- (a) Compute the area of surface of revolution obtained by rotating the curve $y = \sqrt{4 - x^2}$ around the x -axis.
- (b) Do the same for the curve $y = 1 - |x|$, $-1 \leq x \leq 1$.

Problem 2. The curve

$$y = \frac{e^x + e^{-x}}{2}, \quad -1 \leq x \leq 1,$$

is graphed below. Find the surface area of the solid of revolution obtained by rotating this curve about the x -axis.



Problem 3. Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

- a) $x = 1 - t^2$, $y = 2t - t^2$, $-1 \leq t \leq 2$.
- b) $x = 2^t - t$, $y = 2^{-t} + t$, $-3 \leq t \leq 3$.

Problem 4.

1. Eliminate the parameter to find a Cartesian equation of the curve.
2. Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

- a) $x = 3 \cos t$, $y = 3 \sin t$, $0 \leq t \leq \pi$.
- b) $x = \cos \theta$, $y = \sec^2 \theta$, $0 \leq \theta < \frac{\pi}{2}$.

Problem 5. Find parametric equations for the position of a particle moving along a circle as described.

The particle travels clockwise around a circle centered at the origin with radius 5 and completes a revolution in 4π seconds.

Problem 6. Find parametric equations to represent the line segment from $(-2, 7)$ to $(3, -1)$.

Problem 7. The position of a red particle at time t is given by

$$x = t + 5, \quad y = t^2 + 4t + 6$$

and the position of a blue particle is given by

$$x = 2t + 1, \quad y = 2t + 6.$$

a) Graph the paths of both particles. At how many points do the graphs intersect? Do the particles collide? If so, find the collision points.

b) If the equations of the blue particles are

$$x = 2t + 4, \quad y = 2t + 9,$$

then does red and blue particles collide?