Math 162 - Spring 2025
Workshop 10
March 7 - March 11
11.8. Power series, 11.9. Representations of functions as power series
11.10. Taylor and Maclaurin series, 11.11. Applications of Taylor polynomials.

Problem 1. Find the radius of convergence and interval of convergence of the following series

a) $\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{\sqrt{n}} x^n$

b) $\sum_{n=1}^{\infty} \frac{n}{2^n(n^2+1)} x^n$

c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n} (x-1)^n$

d) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n} (x+6)^n$.

Problem 2. Find a power series representation for the function and determine the interval of convergence.

a) $f(x) = \frac{x}{2x^2+1}$.

b) $f(x) = \frac{x}{(1+4x)^2}$.

Problem 3. Evaluate the indefinite integrals as power series and determine the radius of convergence:

a) $\int \frac{t}{1+t^3}\, dt$.

b) $\int \frac{\tan^{-1} x}{x}\, dx$.

Problem 4. Find the Taylor series for $f(x)$ centered at the given value of a. Find the radius of convergence.

a) $f(x) = \ln x$, centered at $a = 2$.

b) $f(x) = x^6 - x^4 + 2$, centered at $a = -2$.

Problem 5. Use the Maclaurin series from Table 1 to find the series for the following functions:

a) $f(x) = x\cos(2x)$.

b) $f(x) = x\cos\left(\frac{1}{2}x^2\right)$.

c) $f(x) = e^{3x} - e^{2x}$.

d) $f(x) = x^2 \ln(1+x^3)$.

Problem 6. Find the function represented by the given power series:

a) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$.

b) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{4n}}{n}$.

Problem 7. Approximate $f(x)$ by a Taylor polynomial of degree n at the number a. Use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the given interval.

$$f(x) = x^{2/3}, a = 1, n = 3, 0.8 \le x \le 1.2$$