

Math 162 - Spring 2025

Workshop 12

April 21-25

Alternating Series Test and Root and Ratio Tests

Problem 1. Consider the tests for convergence we have learned so far: Divergence Test, Integral Test, p -test, Direct Comparison Test, Limit Comparison Test, Alternating Series Test, Ratio Test, Root Test. Quickly review what each of them say. Think about and discuss: for which series would each test be best applied?

Problem 2. For each of the following series, use the alternating series test, if possible, to determine convergence of the following series. If the alternating series test does not apply, explain why.

1. $\sum_{n=1}^{\infty} (-1)^n n e^{-n}$

2. $\sum_{n=1}^{\infty} (-1)^n \frac{2n^2}{5n^2 + 2n + 1}$

3. $\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n!}$

4. $\sum_{n=1}^{\infty} \sin\left(\frac{\pi}{n}\right)$

Problem 3. Determine whether ratio test or root test would be preferable for the following problems (or if they are equally as good). Then use the appropriate test to determine if the series are absolutely convergent, conditionally convergent, or divergent.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^4}$

2. $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

3. $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$

4. $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

5. $\sum_{n=1}^{\infty} \frac{n!}{100^n}$