Math 162 - Spring 2025
Workshop 10
March 7 - March 11
11.1.More on sequences, 11.2 Series.

Problem 1. Consider the sequence defined by:

$$a_1 = 1, \quad a_{n+1} = 3 - \frac{1}{a_n}.$$

- Show that the sequence is increasing.
- Show that the sequence is bounded above by 3.
- Deduce that $\{a_n\}$ is convergent and find its limit.

Problem 2. Determine whether the sequence converges or diverges. If it converges, find the limit.

a) $a_n = \frac{4^n}{1+9^n}$.

b) $a_n = \cos\left(\frac{n\pi}{n+1}\right)$.

Problem 3. Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

a) $\sum_{n=1}^{\infty}\left(\frac{1}{n+2} - \frac{1}{n}\right)$.

b) $\sum_{n=4}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$.

Problem 4. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

a) $\sum_{n=1}^{\infty} \frac{e^{2n}}{6^{n-1}}$.

b) $\sum_{n=1}^{\infty} \frac{6 \cdot 2^{2n-1}}{3^n}$.

c) $2 + 0.5 + 0.125 + 0.03125 + \cdots$

Problem 5. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

a) $\sum_{n=1}^{\infty}\left(\frac{2+n}{1-2n}\right)$.

b) $\sum_{k=1}^{\infty} \frac{k^2}{k^2-2k+5}$.

c) $\sum_{n=1}^{\infty}\left(\frac{3}{5^n} + \frac{2}{n}\right)$.

d) $\sum_{n=1}^{\infty}\left(\frac{1}{e^n} + \frac{1}{n(n+1)}\right)$.