Math 162: Calculus IIA

Second Midterm Exam

October 31, 2024

NAME (please print legibly): ____________________

Your University ID Number: ____________________

Your University email ____________________

Indicate your instructor with a check in the box:

Nathanael Grand	MW 9:00 - 10:15 AM	
Doug Ravenel	MW 10:25 - 11:40 AM	
Peter Oberly	MW 12:30 - 1:45 PM	
Peter Oberly	MW 3:25 - 4:40 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ____________________

- **The presence of calculators, cell phones, smart watches, or other electronic devices at this exam is strictly forbidden. If you have your phone with you, you must turn it into a proctor OR turn your phone off and stow it in your backpack. Failure to do so will be treated as an academic honesty violation.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the space provided at the bottom of each page or half page.**
- **You are responsible for checking that this exam has all 12 pages.**

Integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Trigonometric identities:

$$\cos^2(x) + \sin^2(x) = 1 \qquad \sec^2(x) - \tan^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

Derivatives of trig functions.

$$\frac{d\sin x}{dx} = \cos x \qquad \frac{d\tan x}{dx} = \sec^2 x \qquad \frac{d\sec x}{dx} = \sec x\tan x$$

$$\frac{d\cos x}{dx} = -\sin x \qquad \frac{d\cot x}{dx} = -\csc^2 x \qquad \frac{d\csc x}{dx} = -\csc x\cot x$$

Trigonometric substitution tricks for odd powers of secant and even powers of tangent:

$$u = \sec(\theta) + \tan(\theta) \qquad \sec(\theta)d\theta = \frac{du}{u}$$

$$\sec(\theta) = \frac{u^2+1}{2u} \qquad \tan(\theta) = \frac{u^2-1}{2u}$$

Area of surface of revolution in rectangular coordinates $y = f(x)$ with $0 \le a \le x \le b$:

- about the x-axis: $S = \int_a^b 2\pi|f(x)|\sqrt{1+[f'(x)]^2}\,dx$.
- about the y-axis: $S = 2\pi\int_a^b x\sqrt{1+[f'(x)]^2}\,dx$.

Polar coordinate formulas.

$$x = r\cos(\theta) \qquad r^2 = x^2 + y^2$$

$$y = r\sin(\theta) \qquad \tan(\theta) = y/x$$

Note: $\theta = \arctan(y/x)$ when $x > 0$, and $\theta = \arctan(y/x) + \pi$ when $x < 0$.

Changing θ by any multiple of 2π does not change the location of the point.
Changing the sign of r is equivalent to adding π to θ, which is the same as moving the point to one in the opposite direction and the same distance from the origin.

Area in polar cooridinates for $r = f(\theta)$, with $\alpha \leq \theta \leq \beta$:

$$A = \int_{\alpha}^{\beta} \frac{r^2}{2}\, d\theta.$$

Arc length formulas:

- Rectangular coordinates, $y = f(x)$ with $a \leq x \leq b$:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx.$$

- Polar coordinates, $r = f(\theta)$, $\alpha \leq \theta \leq \beta$:

$$L = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta$$

- Parametric equations, $x = x(t), y = y(t)$ with $a \leq t \leq b$:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

1. (20 points) The problem has four parts, each worth 5 points. Each depends on having correct answers to the previous parts. You should not expect to get credit for an answer based on incorrect information.

Consider the integral

$$\int \frac{x^3\,dx}{x^4 - 10x^2 + 9}.$$

(a) Write the denominator of the fraction as a product of linear factors.

ANSWER:

(b) Are there any values of x for which the integrand is not defined? If so, what are they and why?

ANSWER:

(c) Write the integrand as a sum of fractions with constant numerators, which you may denote by letters of the alphabet. YOU NEED NOT FIND THE VALUES OF THESE CONSTANTS.

ANSWER:

(d) Write the original integral in terms of the constants you named in part (c).

ANSWER:

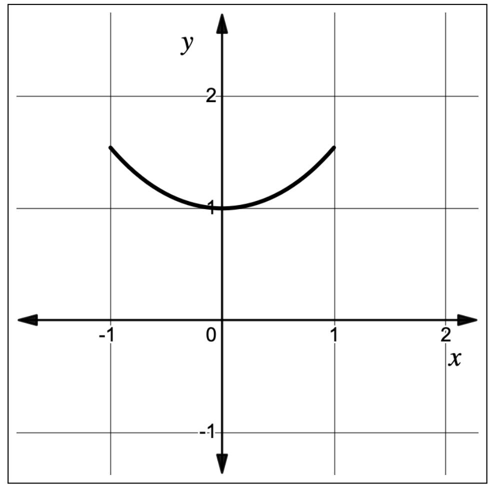
2. (20 points) Find the length of the parametric curve

$$x = 2t - 1, \quad y = t^2 + 6$$

where $0 \leq t \leq 3$.

ANSWER:

3. (20 points)



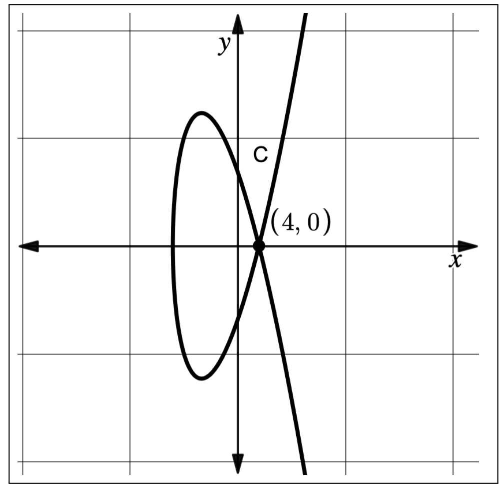
The curve

$$y = \frac{e^x + e^{-x}}{2}, \quad -1 \leq x \leq 1,$$

is graphed to the right. Find the surface area of the solid of revolution obtained by rotating this curve about the x-axis.

ANSWER:

4. (20 points)



The curve C is defined by the parametric equation

$$x = t^2 - 12, \qquad y = t^3 - 16t.$$

Find the equation(s) of the tangent line(s) at the point $(4, 0)$. Note that C intersects itself at $(4, 0)$.

ANSWER:

5. (20 points) Hippopede curves (pictured below) are a family of polar curves, defined by the equations

$$r(\theta) = 2\sqrt{ab - b^2 \sin^2(\theta)} \qquad 0 \leq \theta \leq 2\pi$$

Find the area of the Hippopede curve where $a = 2$ and $b = 1$.

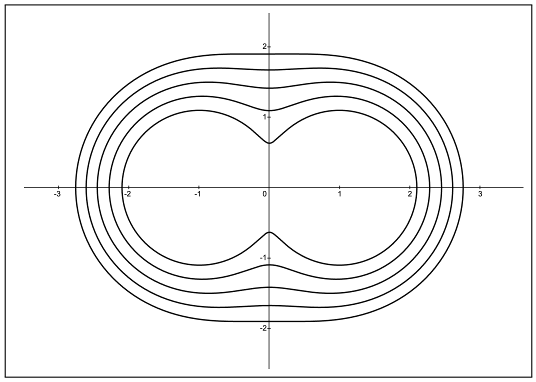


Figure 1: Five different Hippopede curves

ANSWER:

Scratch paper

Scratch paper

Scratch paper