Math 162: Calculus IIA

Second Midterm Exam

November 14, 2023

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Your University email ______________________________

Indicate your instructor with a check in the box:

Firdavs Rakhmonov	MW 9:00 - 10:15 AM	
Doug Ravenel	MW 10:25 - 11:40 AM	
Peter Oberly	MW 12:30 - 1:45 PM	
Sefika Kuzgun	MW 3:25 - 4:40 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ______________________________

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**

- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**

- **Put your answers in the space provided at the bottom of each page or half page.**

- **You are responsible for checking that this exam has all 14 pages.**

Integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Trigonometric identities:

$$\cos^2(x) + \sin^2(x) = 1 \qquad \sec^2(x) - \tan^2(x) = 1$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

Derivatives of trig functions.

$$\frac{d\sin x}{dx} = \cos x \qquad \frac{d\tan x}{dx} = \sec^2 x \qquad \frac{d\sec x}{dx} = \sec x\tan x$$

$$\frac{d\cos x}{dx} = -\sin x \qquad \frac{d\cot x}{dx} = -\csc^2 x \qquad \frac{d\csc x}{dx} = -\csc x\cot x$$

Trigonometric substitution (known in Doug's section as *the rabbit trick.*)) for odd powers of secant and even powers of tangent:

$$u = \sec(\theta) + \tan(\theta) \qquad \sec(\theta)d\theta = \frac{du}{u}$$

$$\sec(\theta) = \frac{u^2+1}{2u} \qquad \tan(\theta) = \frac{u^2-1}{2u}$$

Area of surface of revolution in rectangular coordinates, $y = f(x)$ with $a \le x \le b$

- about the x-axis: $$S = 2\pi\int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$$
- about the y-axis: $$S = 2\pi\int_a^b x\sqrt{1+f'(x)^2}\,dx$$

More formulas for your enjoyment

Polar coordinates

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \begin{cases} \arctan(y/x) & \text{for } x > 0 \\ \pi + \arctan(y/x) & \text{for } x < 0 \\ \pi/2 & \text{for } x = 0 \text{ and } y > 0 \\ 3\pi/2 & \text{for } x = 0 \text{ and } y < 0 \\ \text{undefined} & \text{for } (x, y) = (0, 0) \end{cases}$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

Changing θ by any multiple of 2π does not change the location of the point. Changing the sign of r is equivalent to adding π to θ, which is the same as moving the point to one in the opposite direction and the same distance from the origin.

Area in polar coordinates for $r = f(\theta)$ with $\alpha \le \theta \le \beta$:

$$A = \int_\alpha^\beta \frac{r^2}{2}\, d\theta$$

Arc length formulas

- Rectangular coordinates, $y = f(x)$ with $a \le x \le b$:

$$S = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

- Polar coordinates, $r = f(\theta)$ with $\alpha \le \theta \le \beta$:

$$S = \int_\alpha^\beta \sqrt{r^2 + f'(\theta)^2}\, d\theta$$

- Parametric equations, $x = x(t)$ and $y = y(t)$ with $a \le t \le b$:

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

1. (20 points) Let $a \geq 3$. Determine whether the following improper integral converges. If it does, find its value in terms of a.

$$\int_a^\infty \frac{1}{x\,(\ln x)^2}\,dx$$

ANSWER:

2. (20 points) A curve is described by the following parametric equation:

$$x = t^4 + 5, \quad y = F(t)$$

where $F(t)$ is defined by

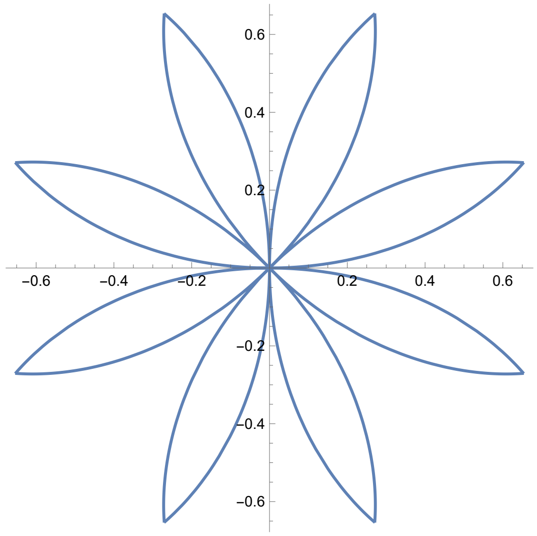
$$F(t) = \int_0^t \sqrt{8u^3 + 1}\, du.$$

Find the length of the curve from $t = 0$ to $t = 1$.

HINT: You will have to use the fundamental theorem of calculus to find dy/dt. Do not attempt to find an antiderivative of $\sqrt{8u^3 + 1}$.

ANSWER:

3. (20 points) Find the area of the region that lies inside the curves $r = \cos 2\theta$ and $r = \sin 2\theta$, which is shown below.



ANSWER:

4. (20 points)

(a) 10 POINTS Compute the area of surface of revolution obtained by rotating the curve $y = \sqrt{4 - x^2}$ around the x-axis.

ANSWER:

(b) 10 POINTS Do the same for the curve $y = 1 - |x|$, $-1 \leq x \leq 1$.

ANSWER:

5. (20 points) Determine if the following sequences are convergent or divergent and explain why. If it is convergent, give its limit.

(a) 10 POINTS

$$\left\{ \frac{n\cos(n)}{n^2+1} \,\middle|\, n \geq 0 \right\}$$

ANSWER:

(b) 10 POINTS

$$\left\{ n^3 \sin\left(\frac{1}{n}\right) \;\middle|\; n \geq 1 \right\}$$

ANSWER:

Scratch paper

Scratch paper

Scratch paper

Scratch paper